Lecture 4

2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, MDCR
- <u>**Attendance at minimum 7 sessions (course or laboratory)**</u>
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - <u>**problems**</u> + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - <u>**all materials/equipments authorized**</u>

2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

Materials

- https://rf-opto.etti.tuiasi.ro

Materials

- RF-OPTO
  - **https://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by **online exam**
  - used at lectures/laboratory

Examen: Logarithmic scales

dB $= 10 \cdot \log_{10}(P_2 / P_1)$

dBm $= 10 \cdot \log_{10}(P / 1\ mW)$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 µW
-30 dBm	= 1 µW
-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

[dBm/Hz] + [dB] = [dBm/Hz]

[x] + [dB] = [x]

Exam

- Complex numbers arithmetic**!!!!**
- $z = a + j \cdot b$; **$j^2 = -1$**

Introduction

~ Microwaves

- Electrical Length (Phase Length)
  - l – physical length
  - E = β·l – electrical Length

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$

$$E = \beta \cdot l = \frac{2\pi}{c_0} \cdot \left(l \cdot f \cdot \sqrt{\varepsilon_r} \right)$$

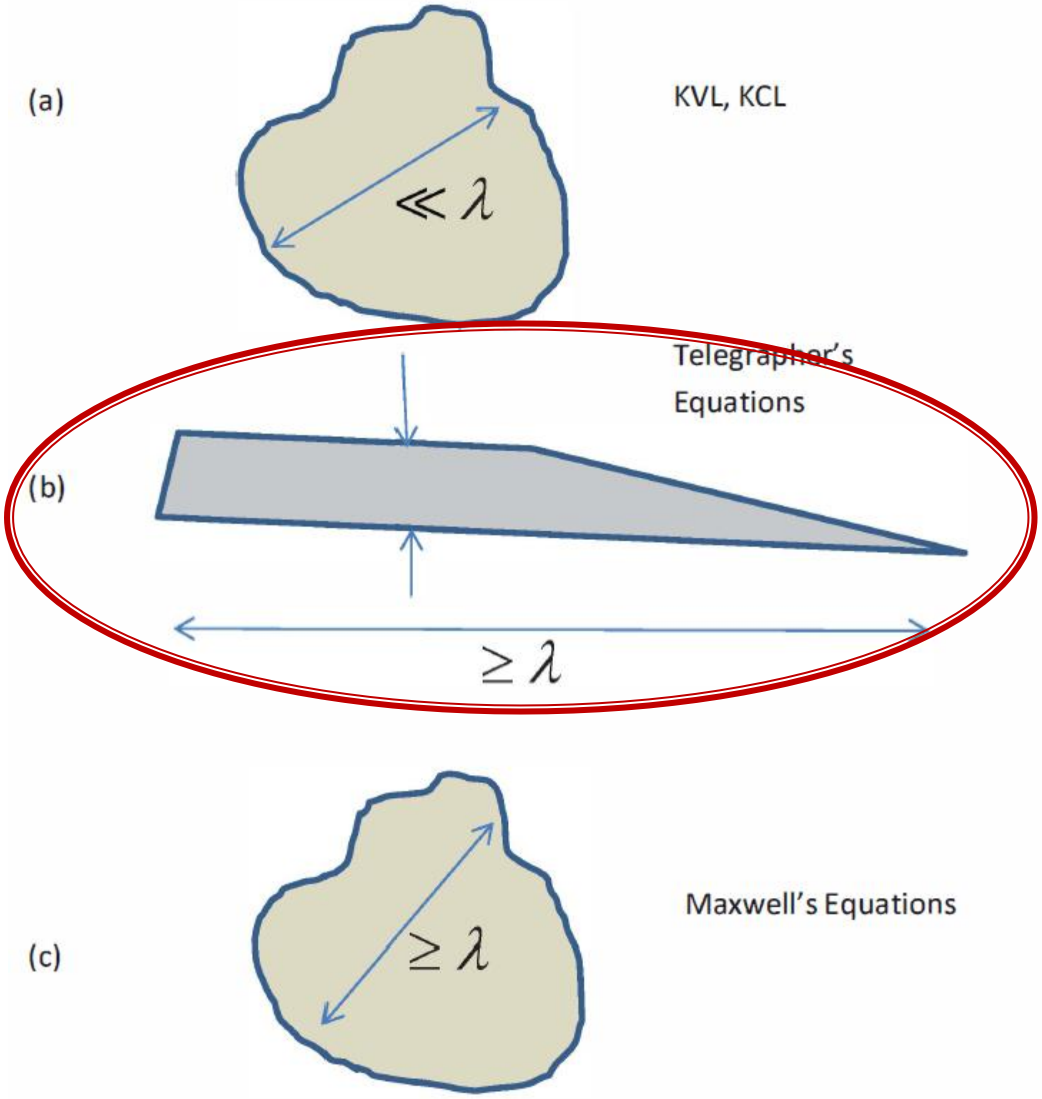
V, I vary
~ useless

- Dependency
  - antenna gain
  - radar cross-section

Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈0 → Kirchhoff
  - E>0 → wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



TEM transmission lines

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

The lossless line

- **Lossless**: R=G=0

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)} = j \cdot \omega \cdot \sqrt{L \cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega \cdot \sqrt{L \cdot C}$$

$$Z_0 = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_0 is **real**

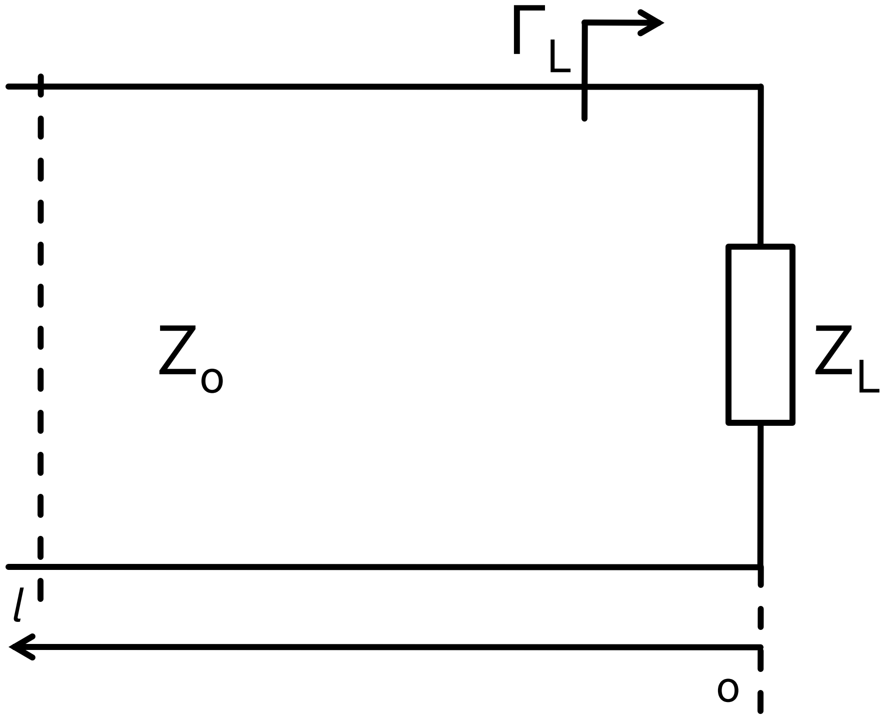
$$V(z) = V_0^+ e^{-j \cdot \beta \cdot z} + V_0^- e^{j \cdot \beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j \cdot \beta \cdot z} - \frac{V_0^-}{Z_0} e^{j \cdot \beta \cdot z}$$

$$\lambda = \frac{2\pi}{\omega \cdot \sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

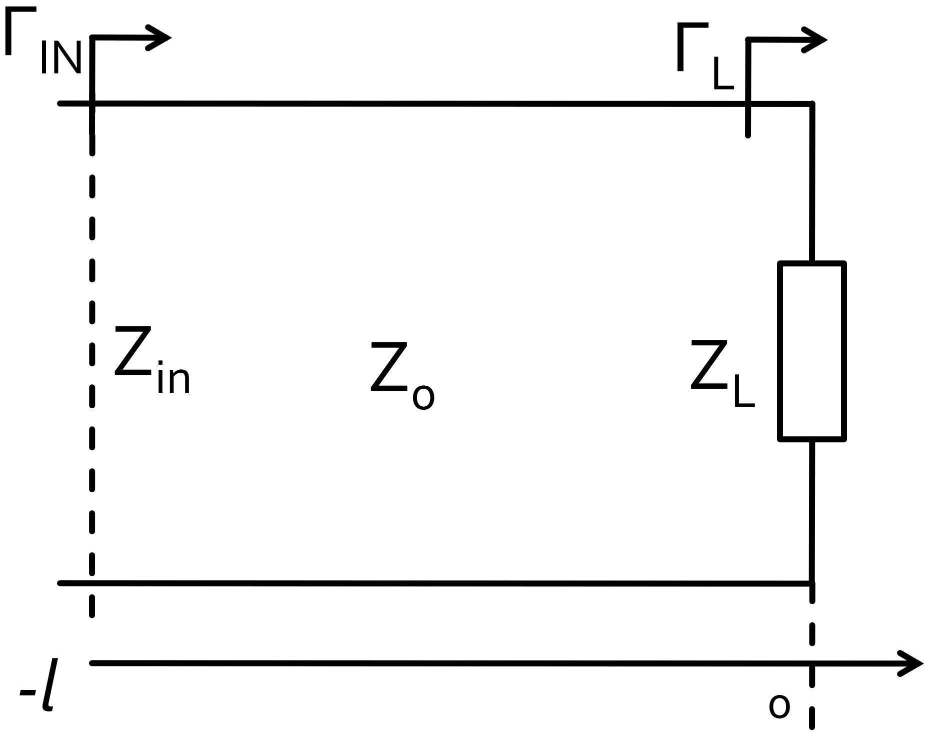
- Z_o real

The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^- \qquad \Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\cdot\beta\cdot l}}{V_0^+ \cdot e^{j\cdot\beta\cdot l}} = \Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma(-l)| = |\Gamma(0)|\cdot\left|e^{-2j\cdot\beta\cdot l}\right| = |\Gamma(0)|$$

$$\boxed{\Gamma_{IN} = \Gamma_L \cdot e^{-2j\cdot\beta\cdot l}} \qquad \boxed{|\Gamma_{IN}| = |\Gamma_L|}$$

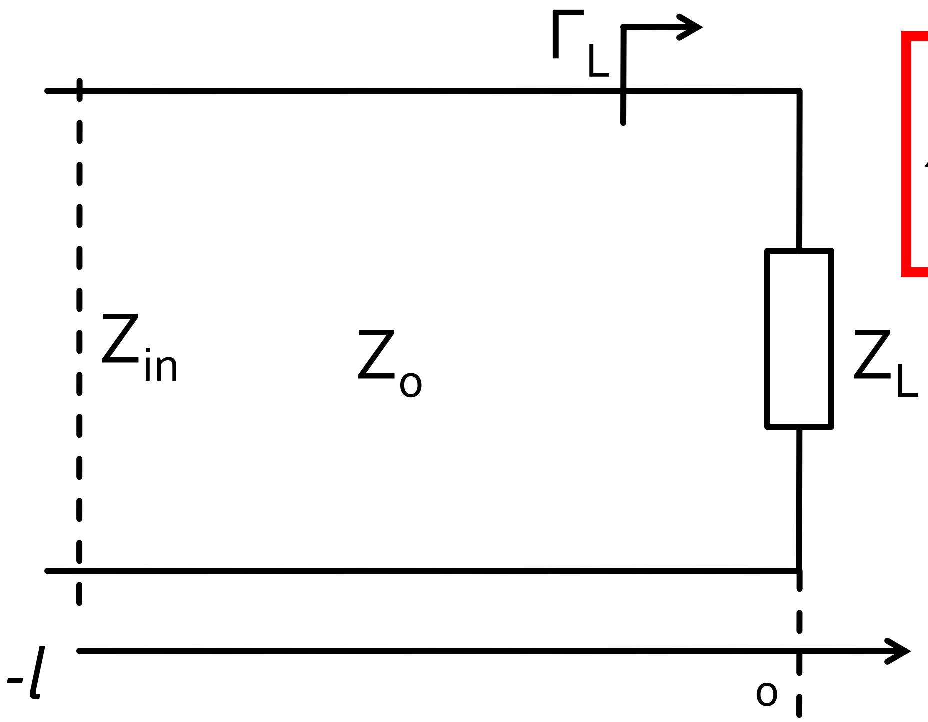
The lossless line

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

- Average power flow is constant along the line
  - (**no** $P_{avg}(\mathbf{z})$)
  - can be measured
- We can use the power to characterize the amplitude of a signal
  - a very “energetic” (basic physics) point of view
  - more power = “more” signal

The lossless line

- input impedance of a length ***l*** of transmission line with characteristic impedance ***Z_o***, loaded with an arbitrary impedance ***Z_L***



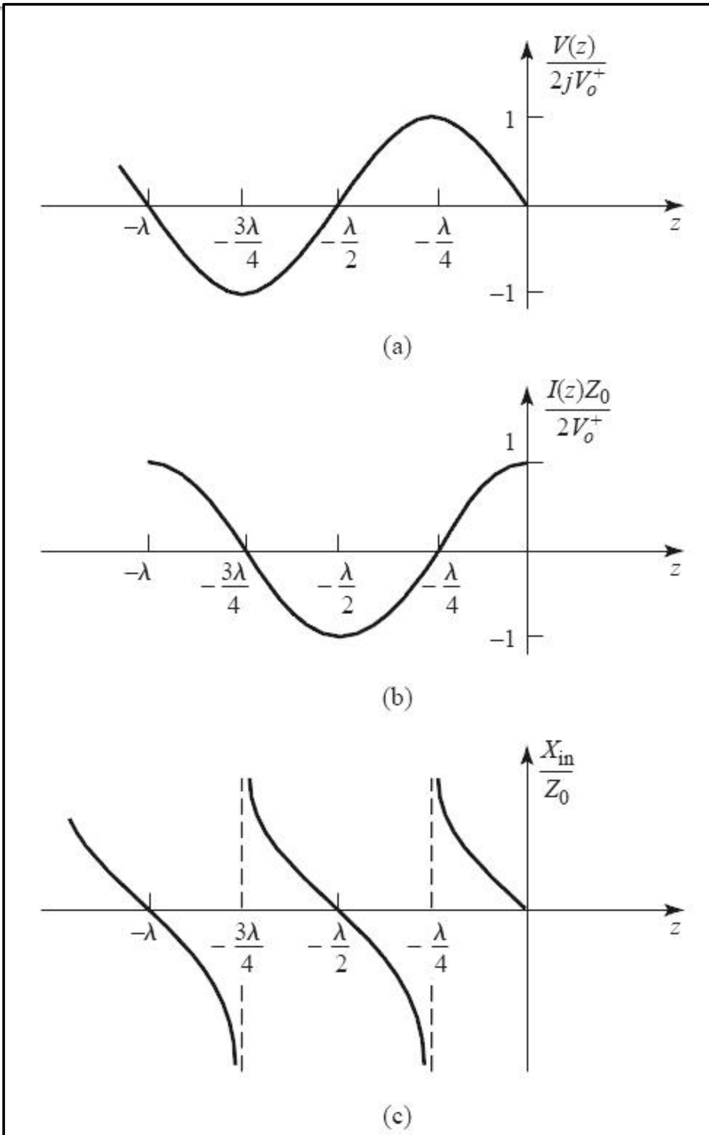
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Short-circuited transmission line

- $Z_L = 0$
- input purely **imaginary** for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

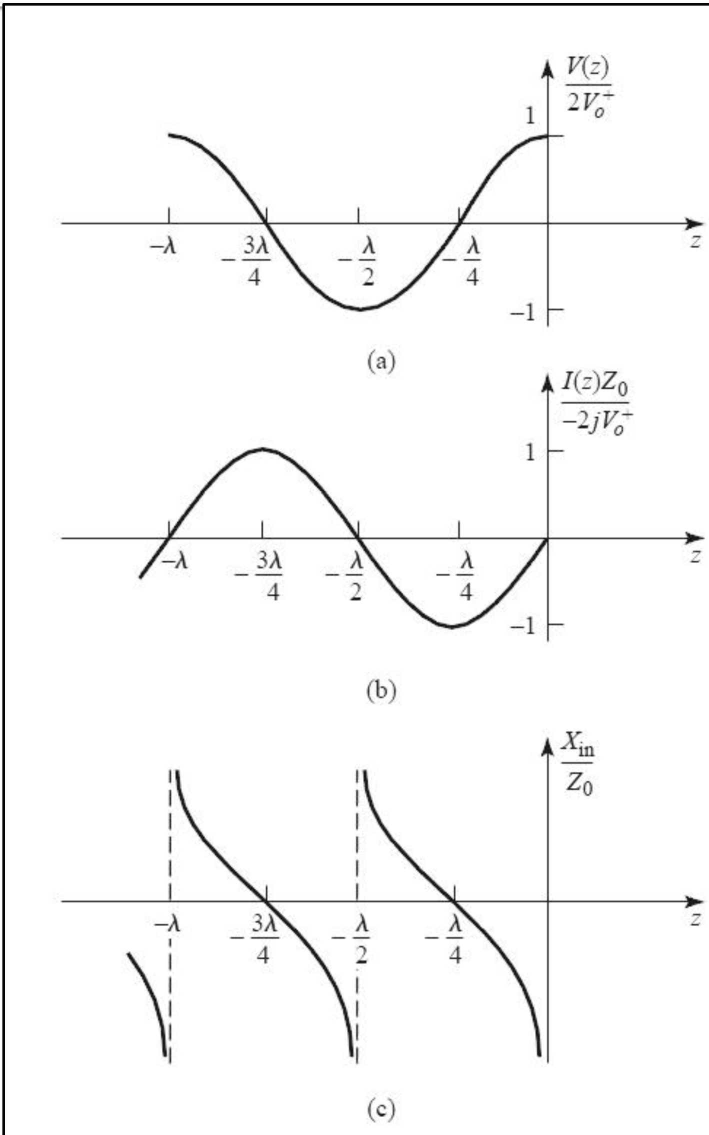


Open-circuited transmission line

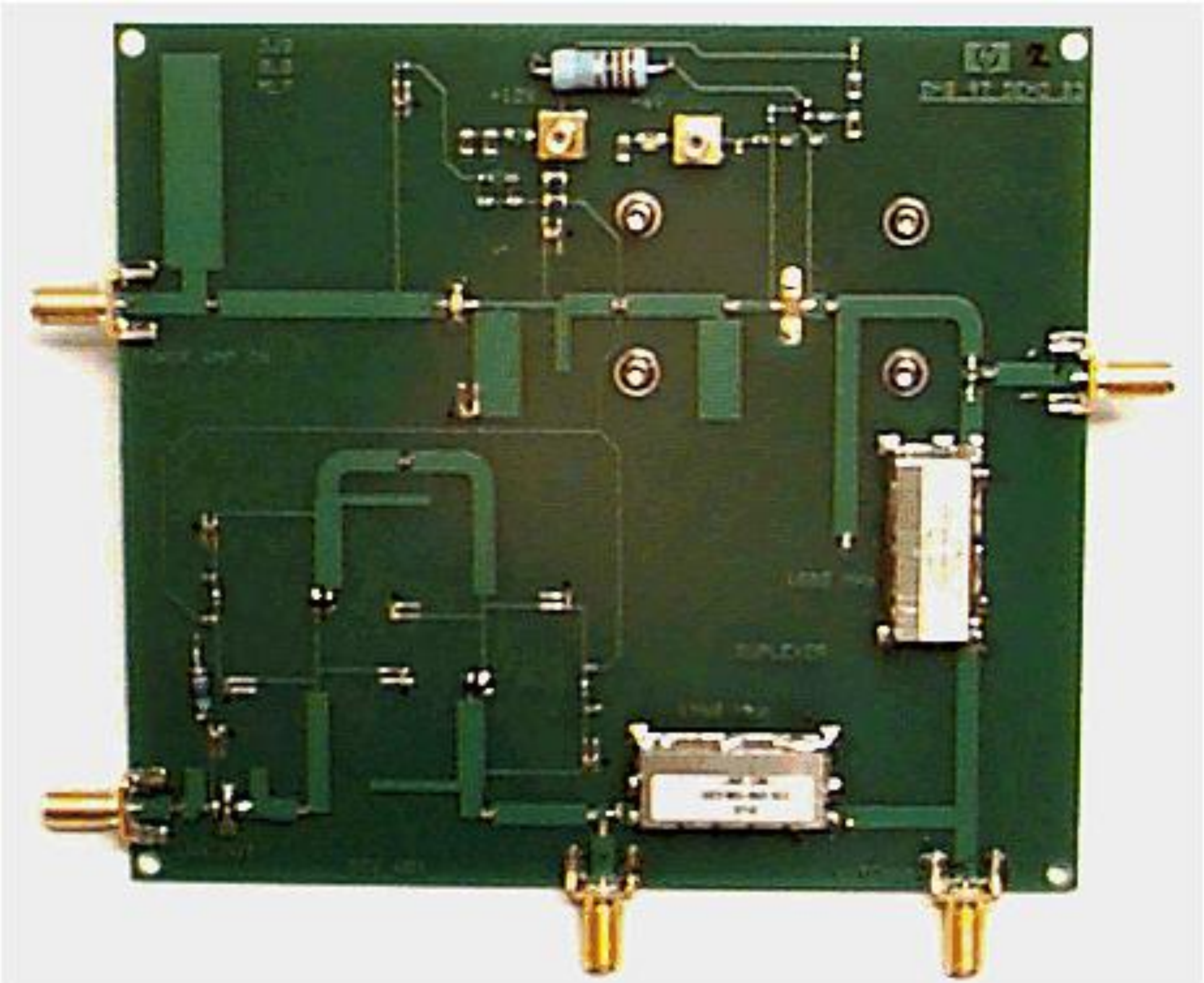
- $Z_L = \infty \rightarrow 1/Z_L = 0$
- input purely **imaginary** for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



Examples



Power transfer

Impedance Matching

Course Topics

- Transmission lines
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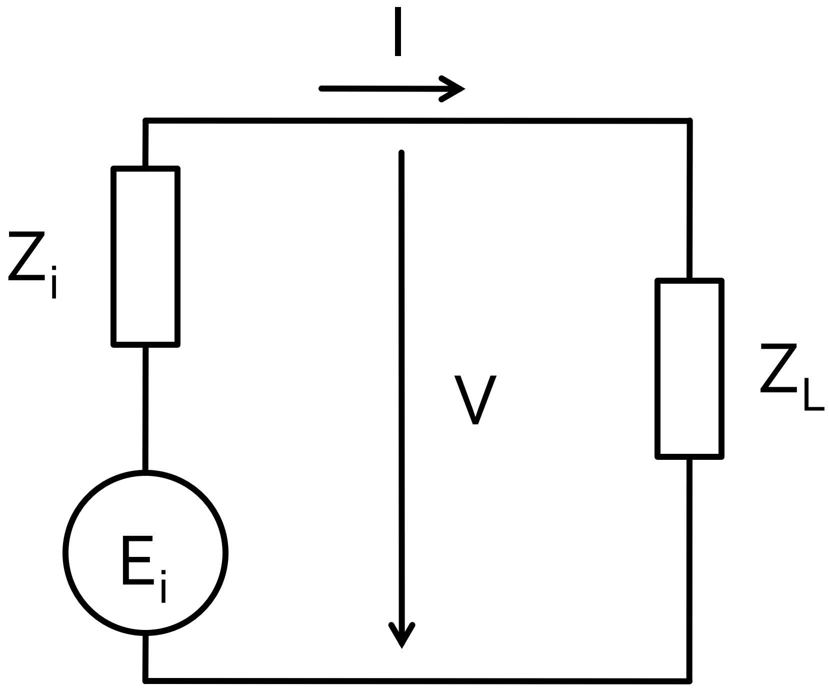
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

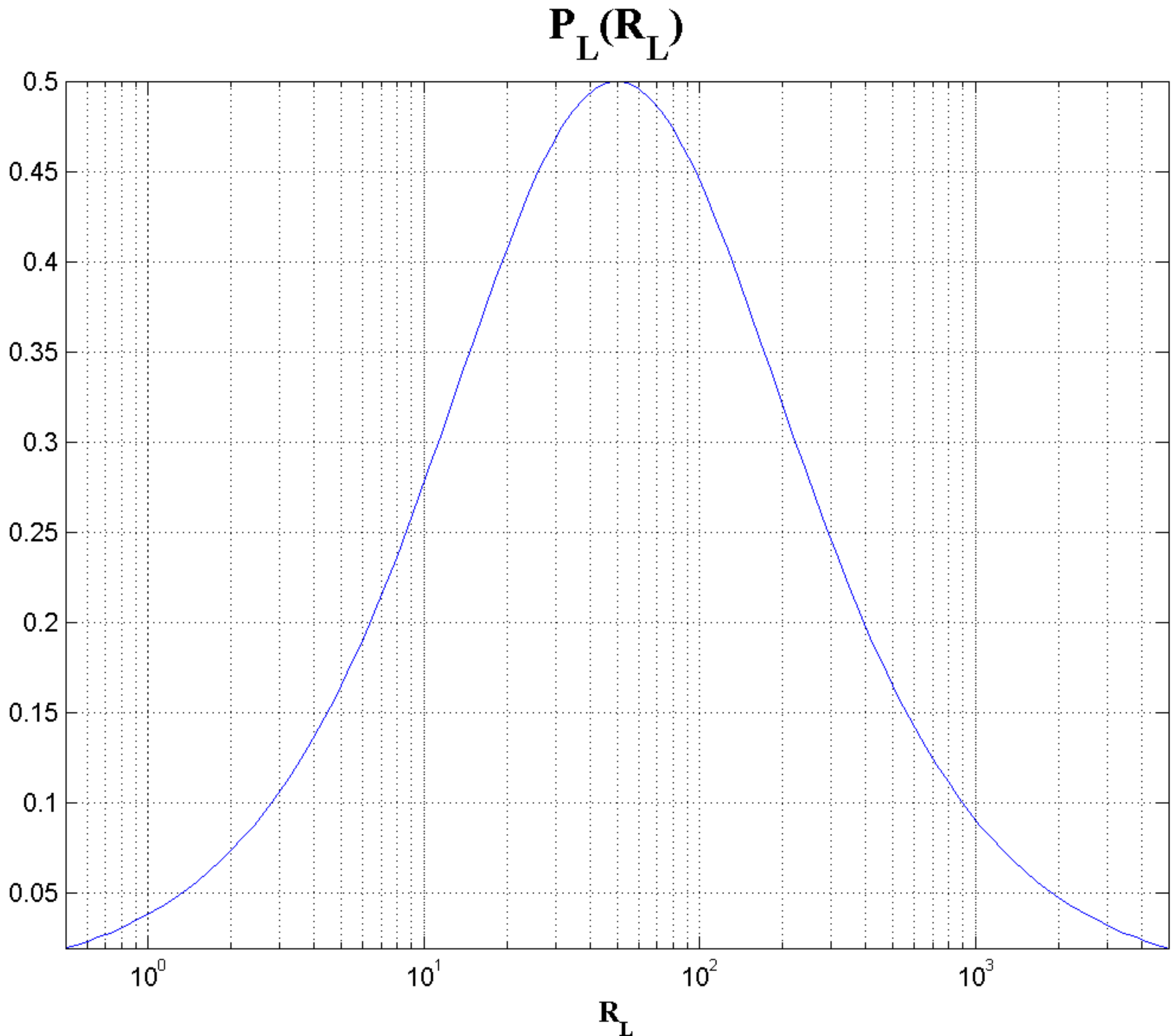
Matching

- Source **matched** to load ?



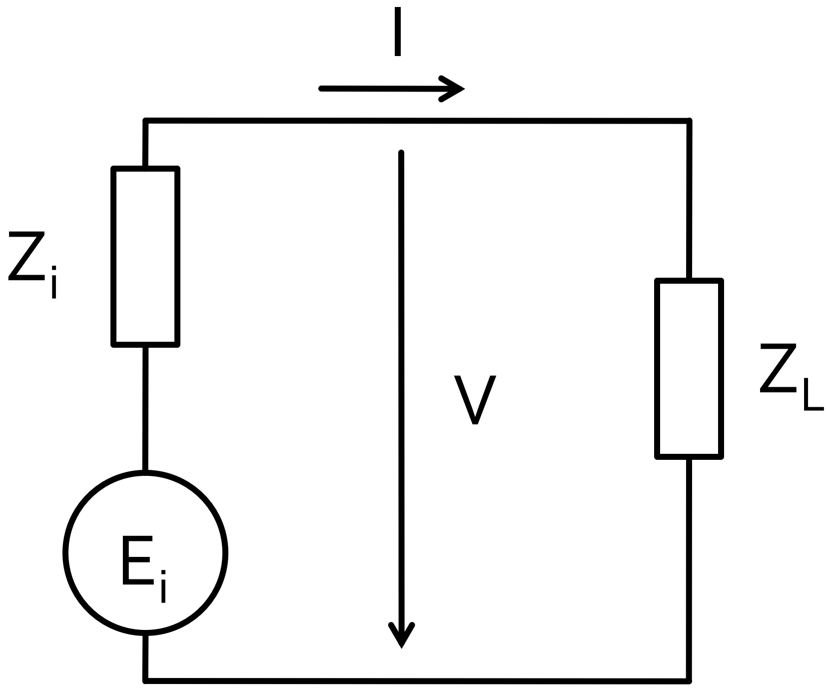
- impedance values ?
- existence of reflections ?

Matching, real impedances



Matching, complex impedances

- Source matched to load



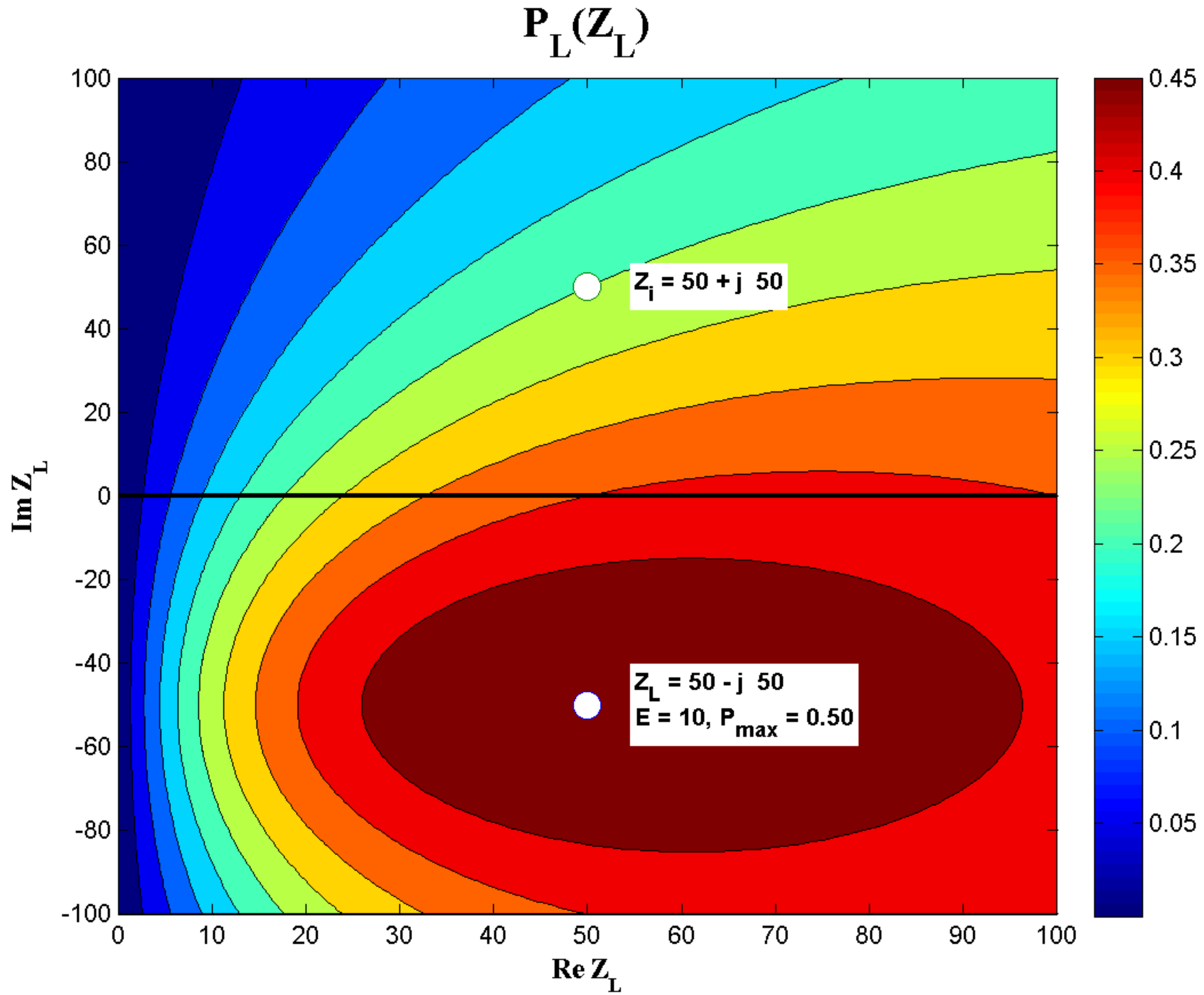
$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \mathrm{Re}\left\{ Z_L \cdot |I|^2 \right\}$$

$$P_L = \mathrm{Re}\left\{ Z_L \right\} \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

Matching, example



Matching , from the point of view of power transmission

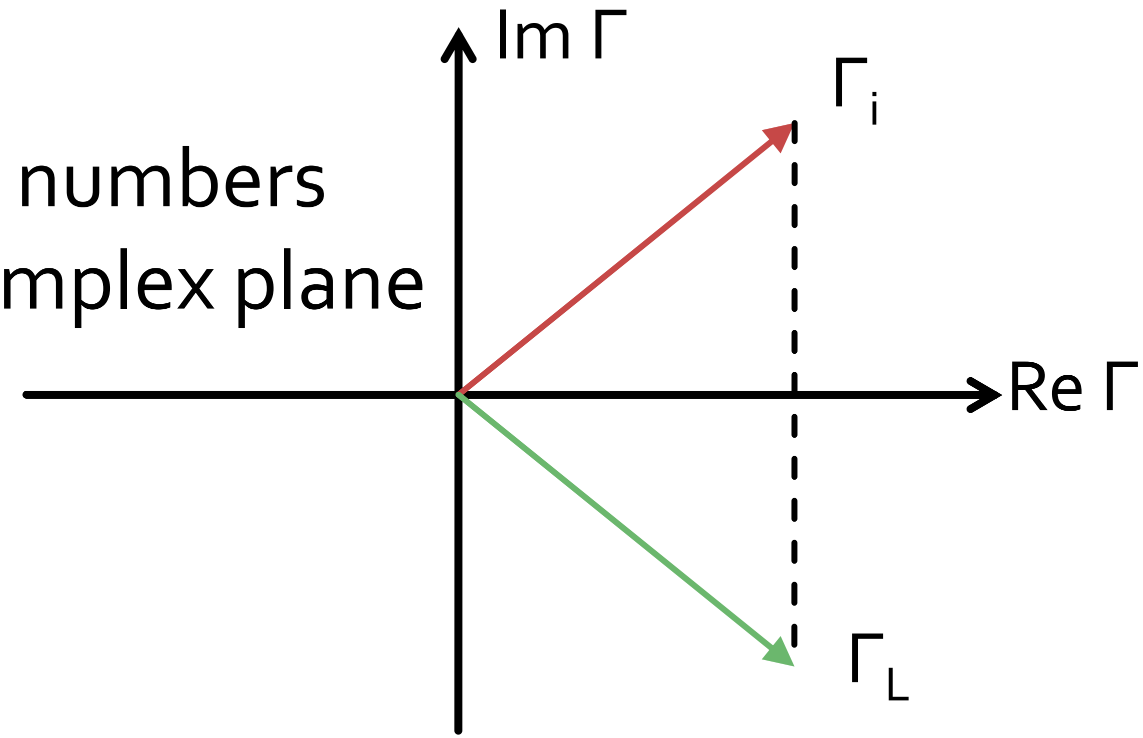
$$Z_L = Z_i^*$$

If we choose a (any) real Zo

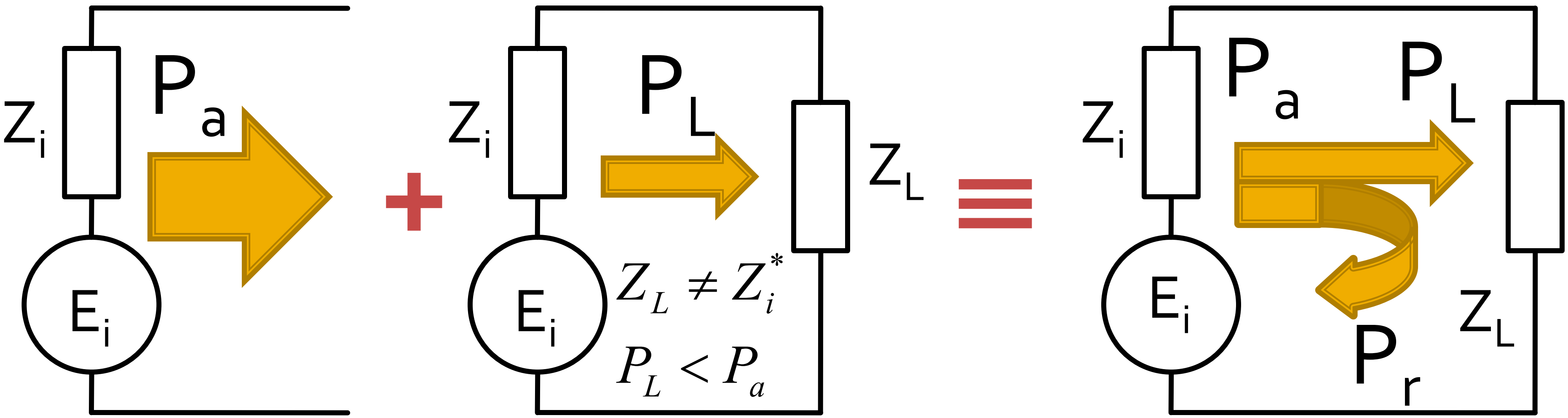
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



Reflection and power / Model



- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a – P_L$
- The power is a **scalar !**

The quarter-wave transformer

Impedance Matching

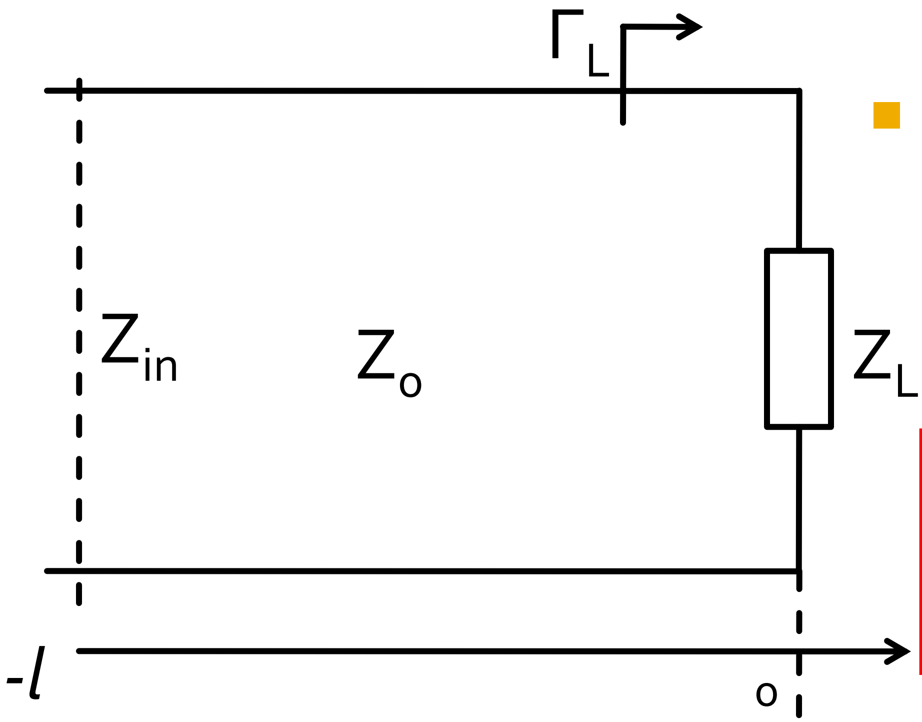
The lossless line, special cases

- $l = k\cdot\lambda/2$ $\quad \beta\cdot l = \frac{2\pi}{\lambda}\cdot l = k\cdot\pi \quad \tan\beta\cdot l = 0$

$$Z_{in} = Z_L$$

- $l = \lambda/4 + k\cdot\lambda/2$ $\quad \beta\cdot l = \frac{\pi}{2} + k\cdot\pi \quad \tan\beta\cdot l \to \infty$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$



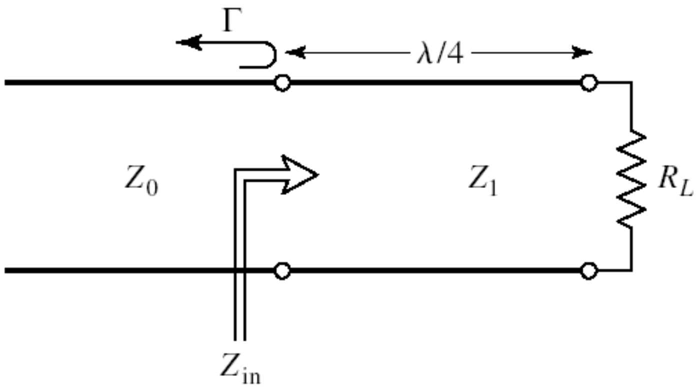
- quarter-wave transformer

$$Z_{in} = Z_0 \cdot \frac{Z_L + j\cdot Z_0\cdot\tan\beta\cdot l}{Z_0 + j\cdot Z_L\cdot\tan\beta\cdot l}$$

The quarter-wave transformer

- Feed line – input line with characteristic impedance Z_o
- **Real** load impedance R_L
- We desire matching the load to the feed line with a second line with the length λ/4 and characteristic impedance Z_1

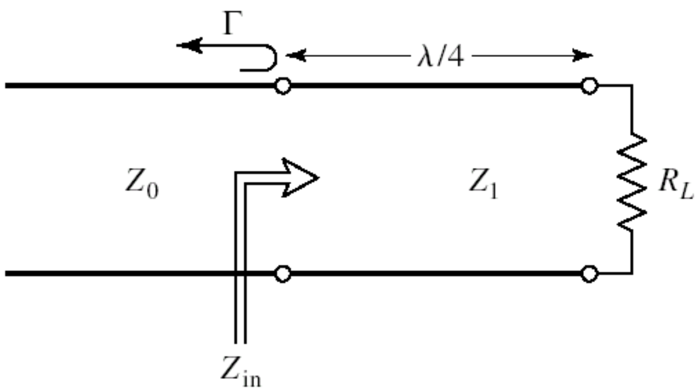
$$Z_{in} = Z_1 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$



$$\Gamma_O = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

The quarter-wave transformer



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \qquad Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \qquad \Gamma_{in} = 0 \quad Z_1 = \sqrt{Z_0 R_L}$$

- In the feed line (Z_o) we have only progressive wave
- In the quarter-wave line (Z_1) we have standing waves

The quarter-wave transformer

- The Multiple-Reflection Viewpoint

$$\Gamma = \Gamma_1 - T_1T_2\Gamma_3 + T_1T_2\Gamma_2\Gamma_3^2 - T_1T_2\Gamma_2^2\Gamma_3^3 + \cdots$$

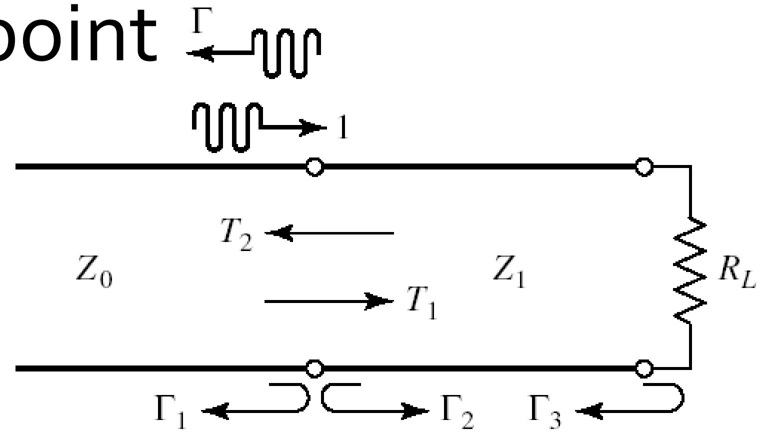
$$= \Gamma_1 - T_1T_2\Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2\Gamma_3)^n.$$

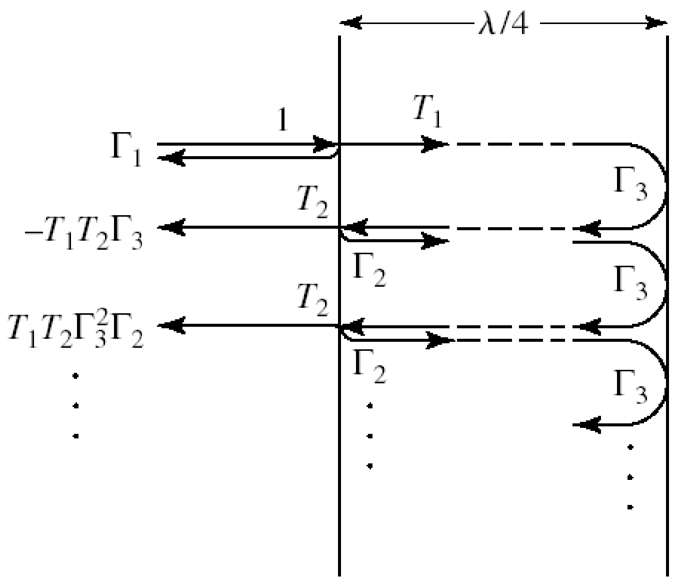
$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$\left.\begin{aligned} T_1 &= \frac{2Z_1}{Z_1 + Z_0}, \\ T_2 &= \frac{2Z_0}{Z_1 + Z_0}. \end{aligned}\right\} \quad T = 1 - \Gamma$$





The quarter-wave transformer

- The Multiple-Reflection Viewpoint

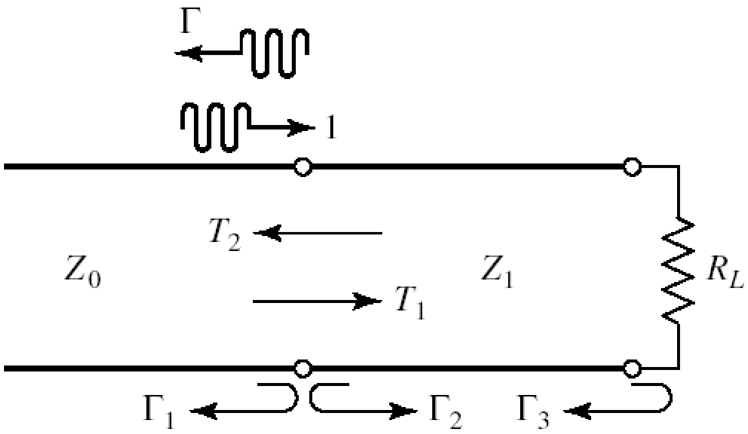
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad \text{for } |x| < 1,$$

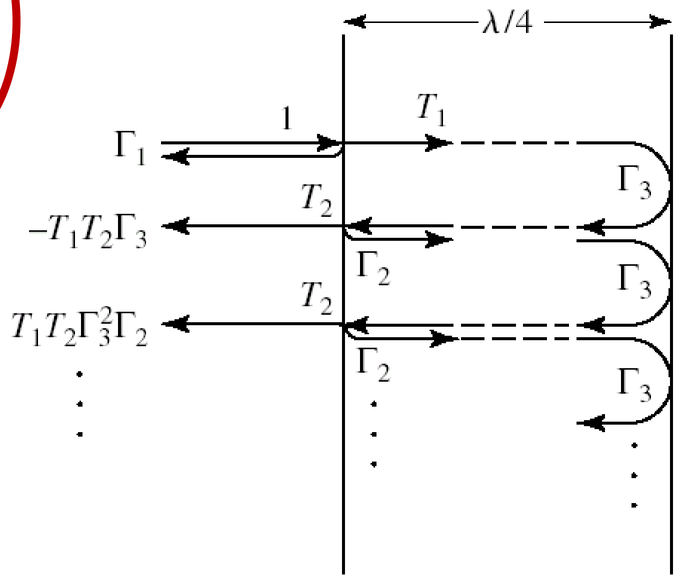
$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}.$$

$$\Gamma_1 - \Gamma_3\left(\Gamma_1^2 + T_1 T_2\right) = \frac{2\left(Z_1^2 - Z_0 R_L\right)}{(Z_1 + Z_0)(R_L + Z_1)},$$

$$\Gamma_1^2 + T_1 T_2 = \frac{\left(Z_1 - Z_0\right)^2}{\left(Z_1 + Z_0\right)^2} + \frac{4 Z_1 Z_0}{\left(Z_1 + Z_0\right)^2} = 1$$

$$\Gamma = 0 \leftrightarrow Z_1^2 - Z_0 \cdot R_L = 0$$





Frequency response

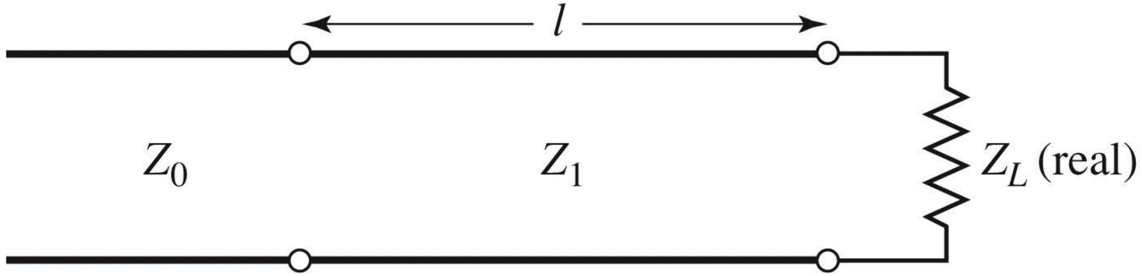


Figure 5.10

$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

- (**only**) at f_o

$$l = \frac{\lambda_0}{4} \qquad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

$$\theta \overset{not}{=} \beta \cdot l \qquad t \overset{not}{=} \tan(\beta \cdot l)$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt\left(Z_1^2 - Z_0 Z_L\right)}{Z_1(Z_L + Z_0) + jt\left(Z_1^2 + Z_0 Z_L\right)}.$$

$$Z_1^2 = Z_0 \cdot Z_L$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}.$$

Frequency response

- matching quality ≡ power reflection coefficient

$$|\Gamma| = \frac{|Z_L - Z_0|}{\left[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L\right]^{1/2}}$$

$$= \frac{1}{\left\{(Z_L + Z_0)^2/(Z_L - Z_0)^2 + [4t^2 Z_0 Z_L/(Z_L - Z_0)^2]\right\}^{1/2}}$$

$$= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] + [4Z_0 Z_L t^2/(Z_L - Z_0)^2]\right\}^{1/2}}$$

$$= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2]\ \sec^2\theta\right\}^{1/2}},$$

$$\sec\theta = \frac{1}{\cos\theta} \rightarrow$$

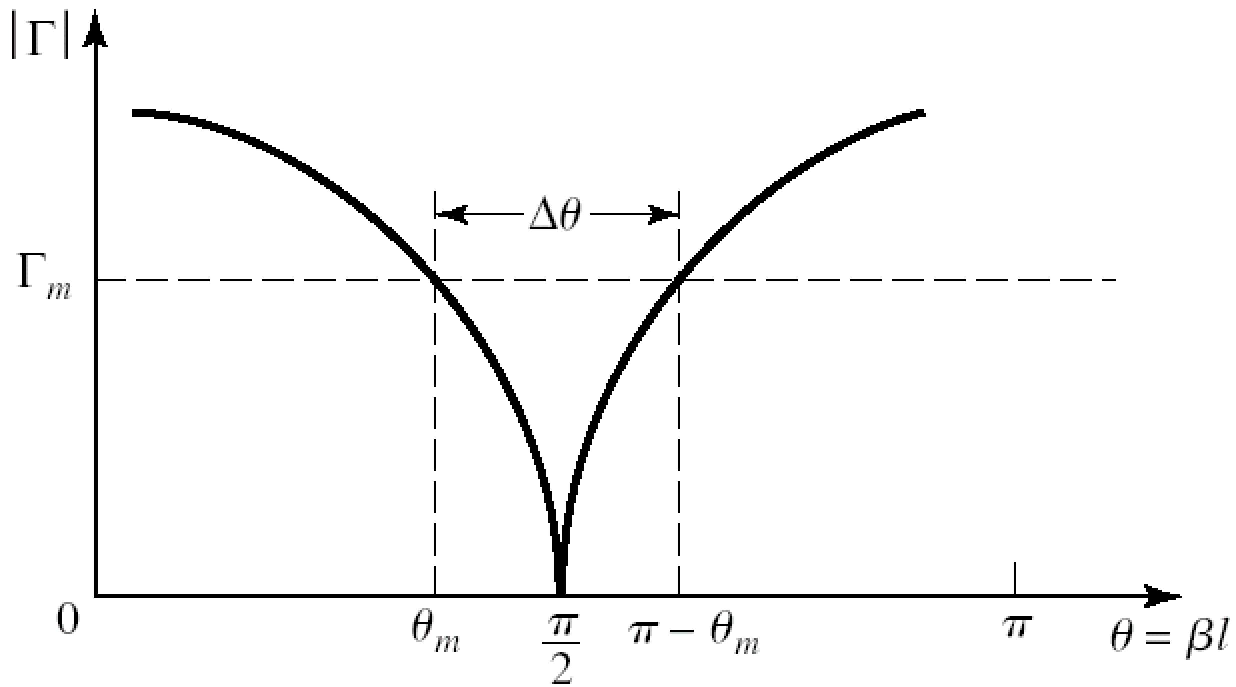
$$\sec^2\theta = 1 + \tan^2\theta = 1 + t^2$$

Frequency response

- we assume that the operating frequency is near the design frequency (narrow bandwidth)

$$f \approx f_0 \qquad l \approx \frac{\lambda_0}{4} \qquad \theta \approx \frac{\pi}{2} \qquad \sec^2\theta = 1 + \tan^2\theta >> 1$$

$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos\theta|$$



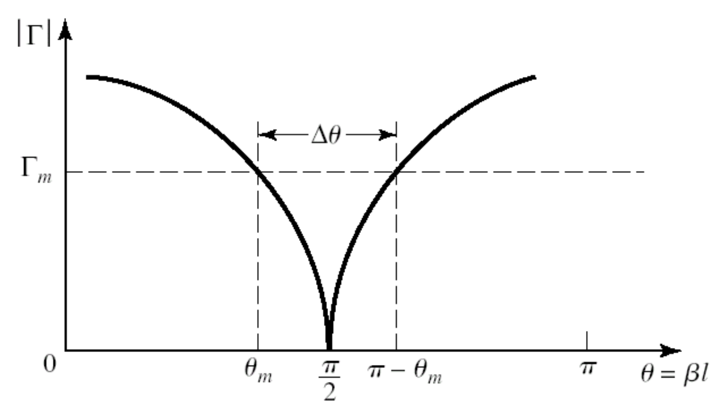
Frequency response

- we set a maximum value Γ_m for an acceptable reflection coefficient magnitude then the bandwidth of the matching transformer, θ_m

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0}\sec\theta_m\right)^2,$$

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$



- for TEM lines

$$\theta = \beta \cdot l = \beta \cdot \frac{\lambda_0}{4} = \frac{2\pi \cdot f}{v_f} \cdot \frac{1}{4} \cdot \frac{v_f}{f_0} = \frac{\pi \cdot f}{2 f_0} \qquad f_m = \frac{2 \cdot \theta_m \cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2 \cdot (f_0 - f_m)}{f_0} = 2 - \frac{4 \cdot \theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \cdot \frac{2\sqrt{Z_0 \cdot Z_L}}{|Z_L - Z_0|}\right]$$

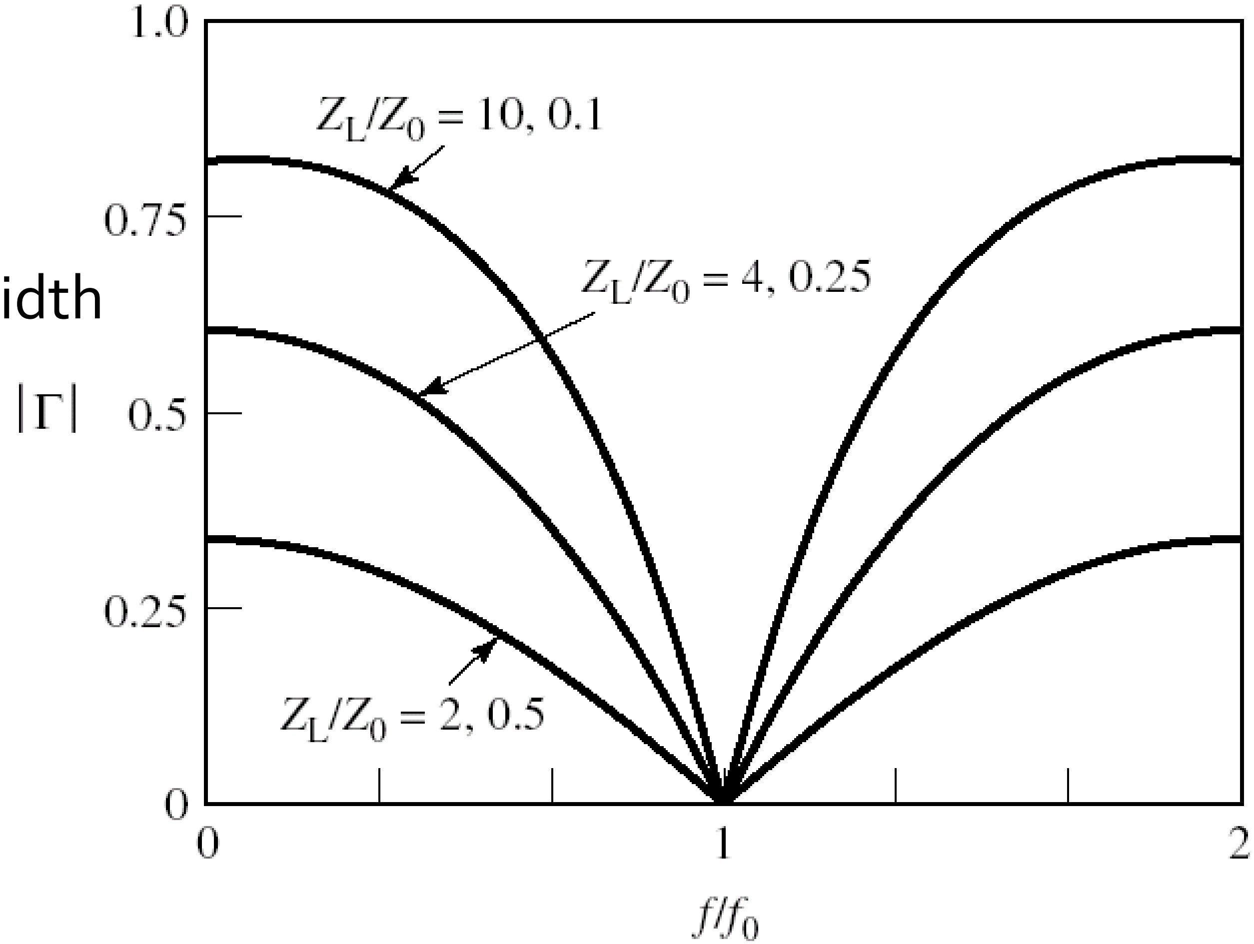
Frequency response

- When non-TEM lines (such as waveguides) are used, the propagation constant is no longer a linear function of frequency, and the wave impedance will be frequency dependent, but in practice the bandwidth of the transformer is often small enough that these complications do not substantially affect the result
- We ignored also the effect of reactances associated with discontinuities when there is a step change in the dimensions of a transmission line ($Z_0 \rightarrow Z_1$). This can often be compensated by making a small adjustment in the length of the matching section

Frequency response

- Bandwidth depends on the initial mismatch

increased bandwidth for smaller load mismatches



Example

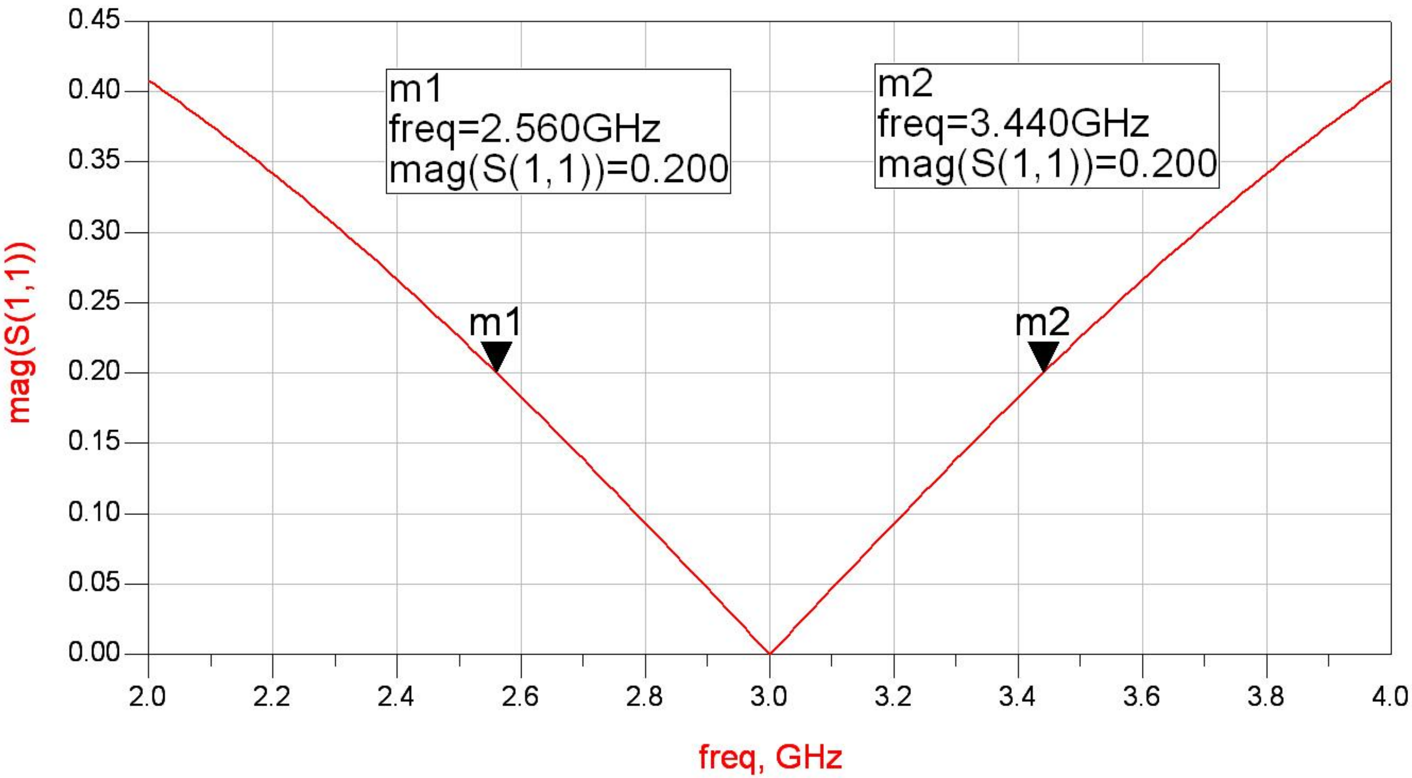
- A quarter-wave matching transformer to match a 10Ω load to a 50 Ω transmission line at f_o=3GHz
  - Determine the percent bandwidth for SWR<1.5

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36\ \Omega, \qquad \Gamma_m = \frac{\mathrm{SWR}-1}{\mathrm{SWR}+1} = \frac{1.5-1}{1.5+1} = 0.2.$$

$$\begin{aligned}\frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] \\ &= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{0.2}{\sqrt{1-(0.2)^2}}\frac{2\sqrt{(50)(10)}}{|10-50|}\right] \\ &= 0.29,\ \text{or } 29\%.\end{aligned}$$

Simulation

- ADS Simulation

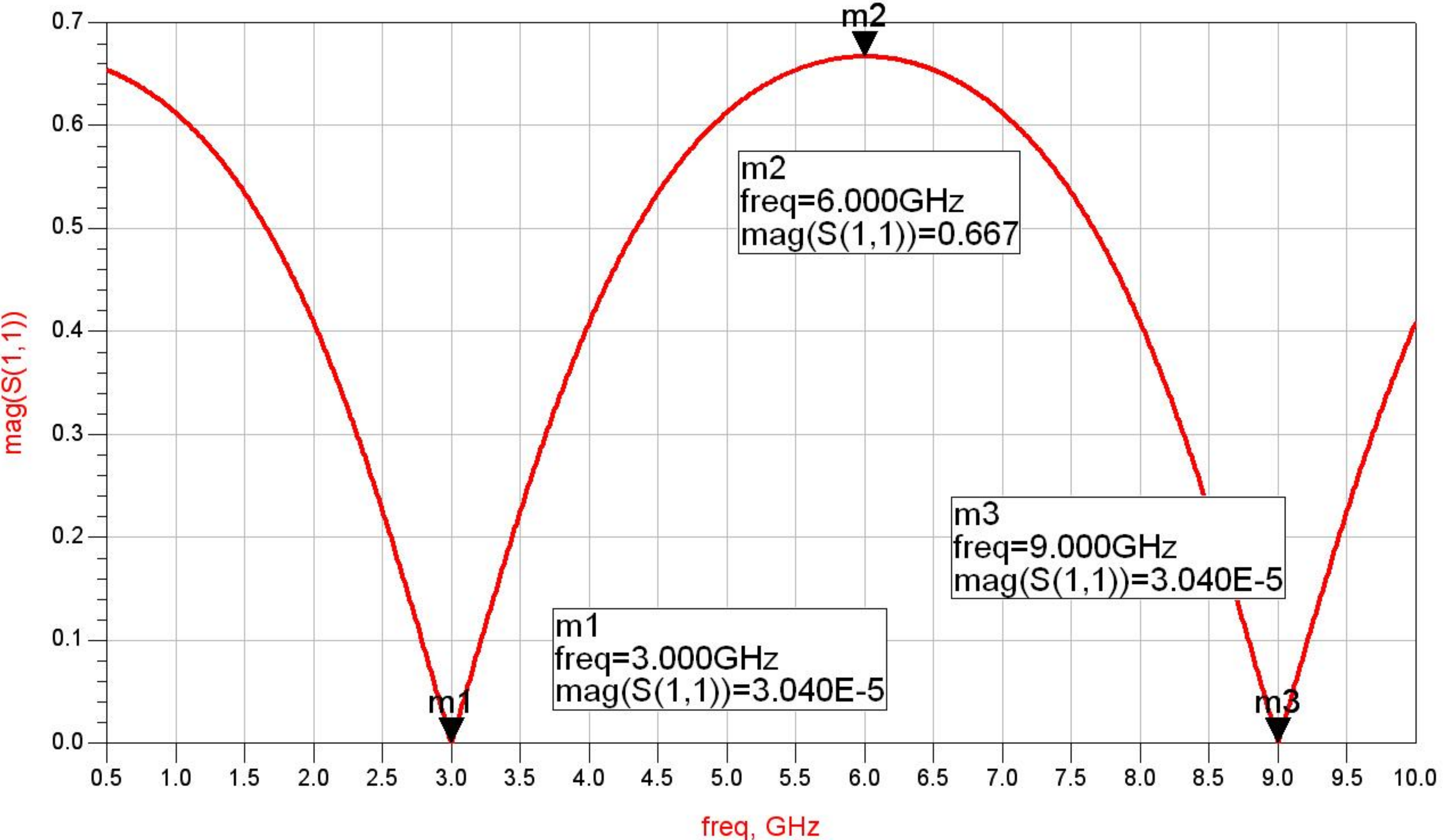


$$\Delta f = 0.88GHz$$

$$\left|\Gamma(3GHz)\right| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

Full bandwidth simulation



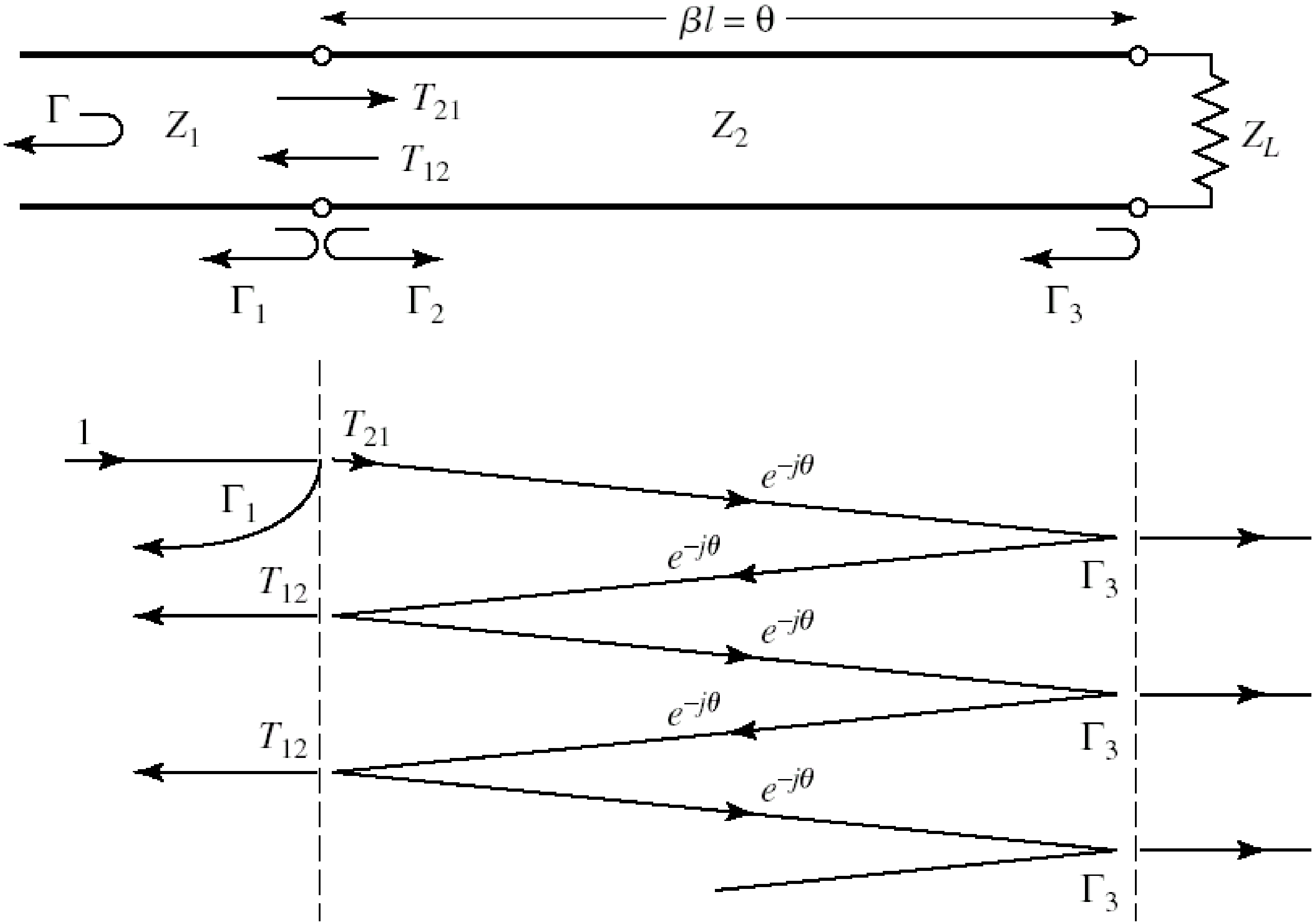
Impedance Matching with Impedance Transformers (Lab 1)

Impedance Matching

Multisection Impedance Transformer

- The quarter-wave transformer can match any real load to any feed line impedance
- If a greater bandwidth for the match is required we must use multiple sections of transmission lines transformers:
  - binomial
  - Chebyshev

The theory of small reflections

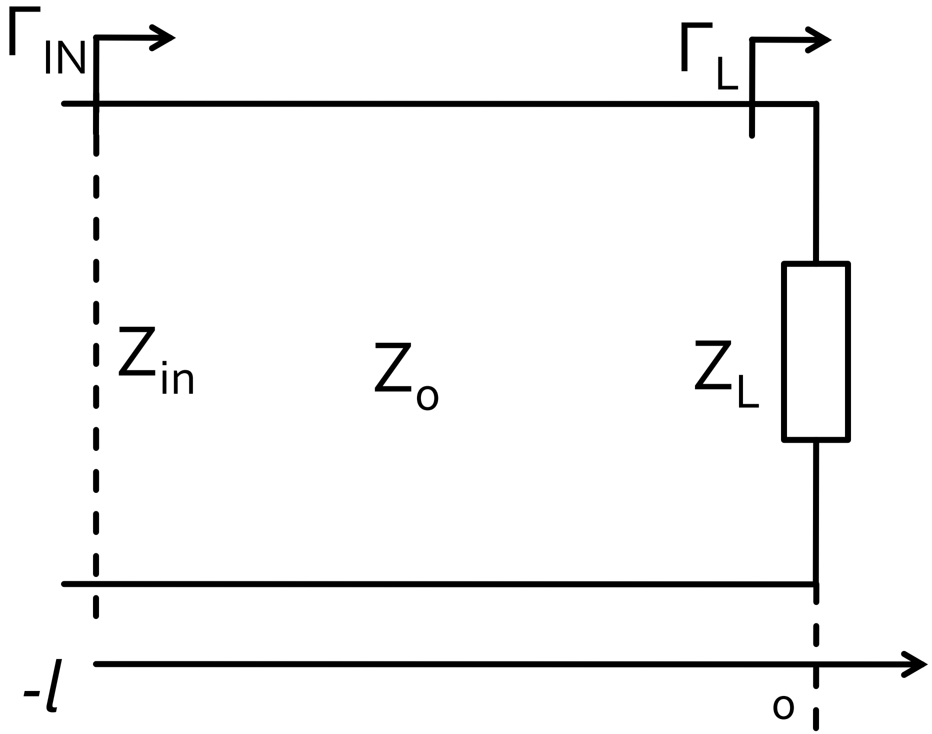


The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^- \qquad \Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\cdot\beta\cdot l}}{V_0^+ \cdot e^{j\cdot\beta\cdot l}} = \Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma(-l)| = |\Gamma(0)|\cdot\left|e^{-2j\cdot\beta\cdot l}\right| = |\Gamma(0)|$$

$$\boxed{\Gamma_{IN} = \Gamma_L \cdot e^{-2j\cdot\beta\cdot l}} \qquad \boxed{|\Gamma_{IN}| = |\Gamma_L|}$$

The theory of small reflections

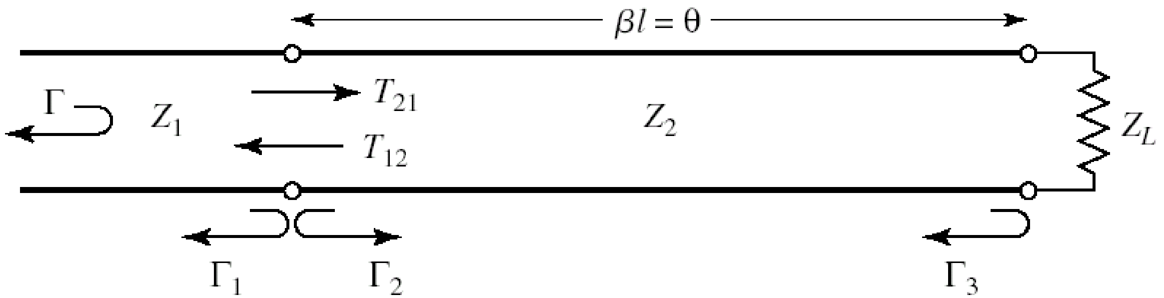
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

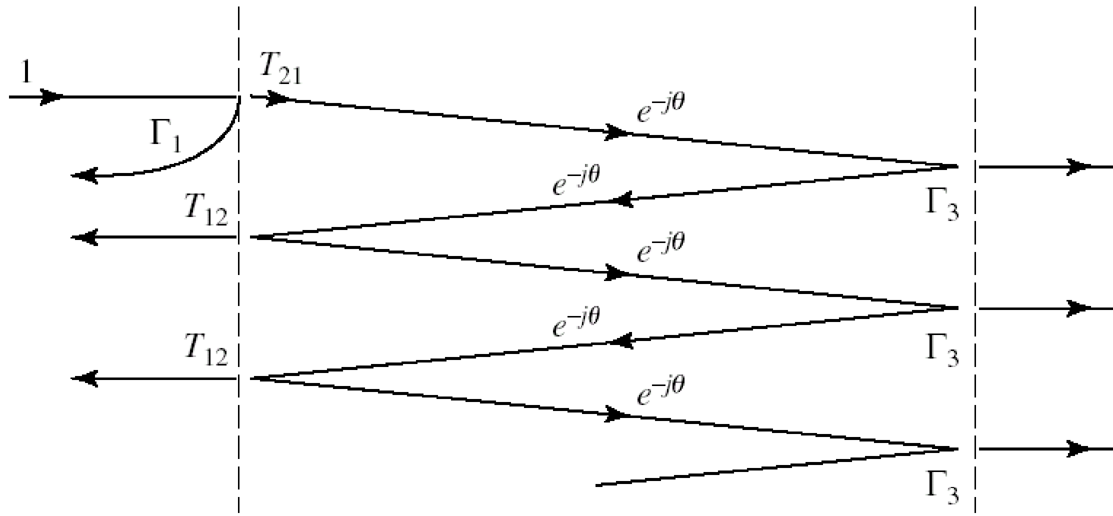
$$\Gamma_2 = -\Gamma_1$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2 \cdot Z_2}{Z_1 + Z_2}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2 \cdot Z_1}{Z_1 + Z_2}$$





$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^2 \cdot \Gamma_2 \cdot e^{-4j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^3 \cdot \Gamma_2^2 \cdot e^{-6j\theta} + \cdots$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

The theory of small reflections

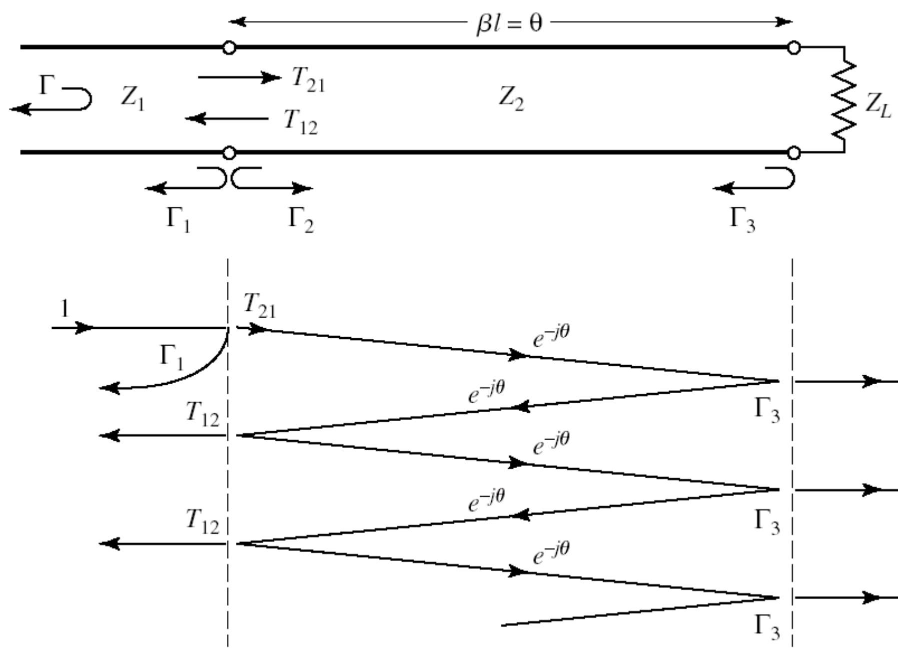
$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

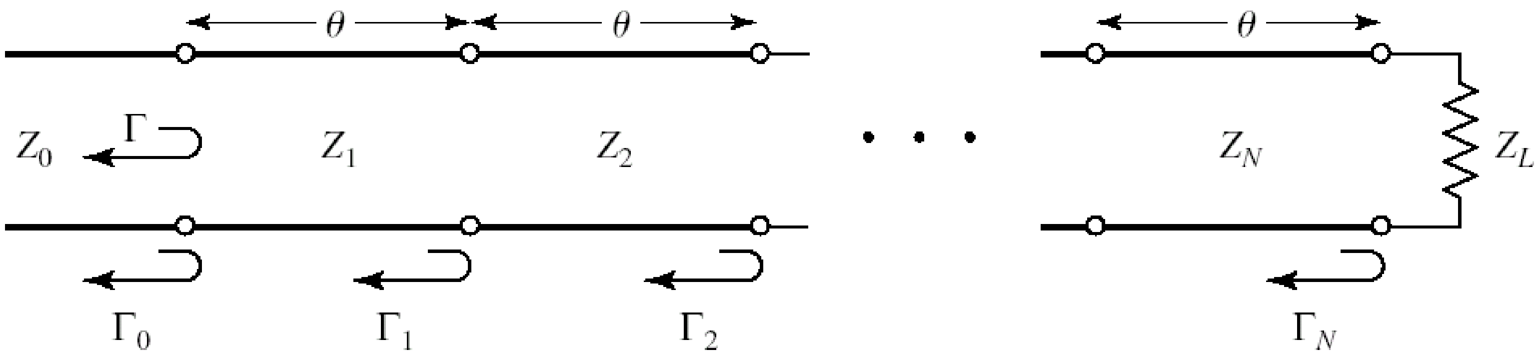
$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}}{1 + \Gamma_1 \cdot \Gamma_3 \cdot e^{-2j\theta}}$$

- If the discontinuities between the impedances $Z_1 \div Z_2$ and $Z_2 \div Z_L$ are small we can approximate

$$\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}$$



Multisection transformers



- We also assume that all impedances **increase or decrease monotonically** across the transformer
- This implies that all reflection coefficients will be real and of the same sign
- Previously, 1 section $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$n = \overline{1, N-1}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

Multisection transformers

- assume that the transformer can be made **symmetrical**

$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2} \cdots$$

- Note that this does **not** imply that the impedances are symmetrical

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \left(e^{jN\theta} + e^{-jN\theta}\right) + \Gamma_1 \cdot \left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + \Gamma_2 \cdot \left(e^{j(N-4)\theta} + e^{-j(N-4)\theta}\right) + \cdots\right]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

last item: $\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad N \text{ even}$ $\qquad \cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad N \text{ odd}$

Multisection transformers

- Input reflection coefficient

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

- we can choose the coefficients so we obtain a desired behavior (of the polynomial)

Binomial multisection transformer

- The response is as flat as possible near the design frequency, also known as **maximally flat**
- For N sections the first N-1 derivatives of the |Γ(θ)| functions are annulled

$$f(x) = A \cdot (1+x)^N$$

$$\Gamma(\theta) = A \cdot \left(1+e^{-2j\theta}\right)^N$$

$$|\Gamma(\theta)| = |A| \cdot \left|e^{-j\theta}\right|^N \cdot \left|e^{j\theta}+e^{-j\theta}\right|^N = 2^N \cdot |A| \cdot |\cos\theta|^N$$

$$l = \frac{\lambda}{4} \Rightarrow \theta = \beta \cdot l = \frac{\pi}{2} \qquad \left|\Gamma\left(\frac{\pi}{2}\right)\right| = 0; \quad \frac{d^n}{d\theta^n}|\Gamma(\theta)|\Big|_{\theta=\frac{\pi}{2}} = 0 \qquad n = \overline{1, N-1}$$

Binomial multisection transformer

- A, θ →0 , 0 length sections, the sections disappear

$$\Gamma(0) = 2^N \cdot A = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Binomial expansion

$$f(x) = (1+x)^N = C_N^0 + C_N^1 \cdot x + \cdots + C_N^n \cdot x^n + \cdots + C_N^N \cdot x^N$$

$$C_N^n = \frac{N!}{(N-n)!\,n!}$$

- Reflection coefficient:

$$\Gamma(\theta) = A \cdot \left(1 + e^{-2j\theta}\right)^N \qquad \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_n = A \cdot C_N^n$$

Binomial multisection transformer

- Manual design procedure

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_n = A \cdot C_N^n$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \cong \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n}$$

$$\ln x \cong 2 \cdot \frac{x-1}{x+1} \quad x \cong 1$$

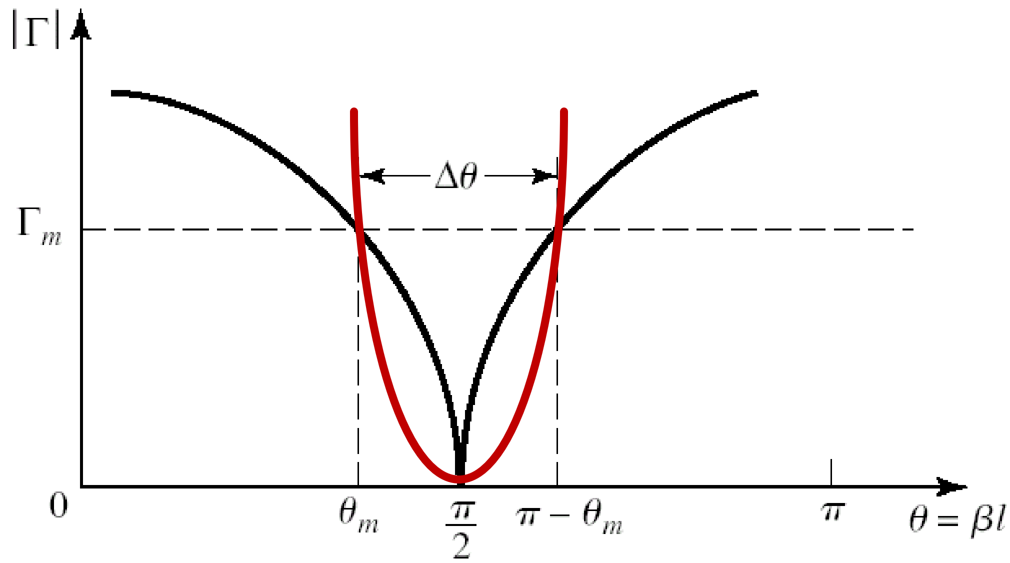
$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n = 2 \cdot A \cdot C_N^n = 2 \cdot 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \cong 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

$$\ln Z_{n+1} \cong \ln Z_n + 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

Binomial multisection transformer

- Bandwidth, Γ_m maximum acceptable value

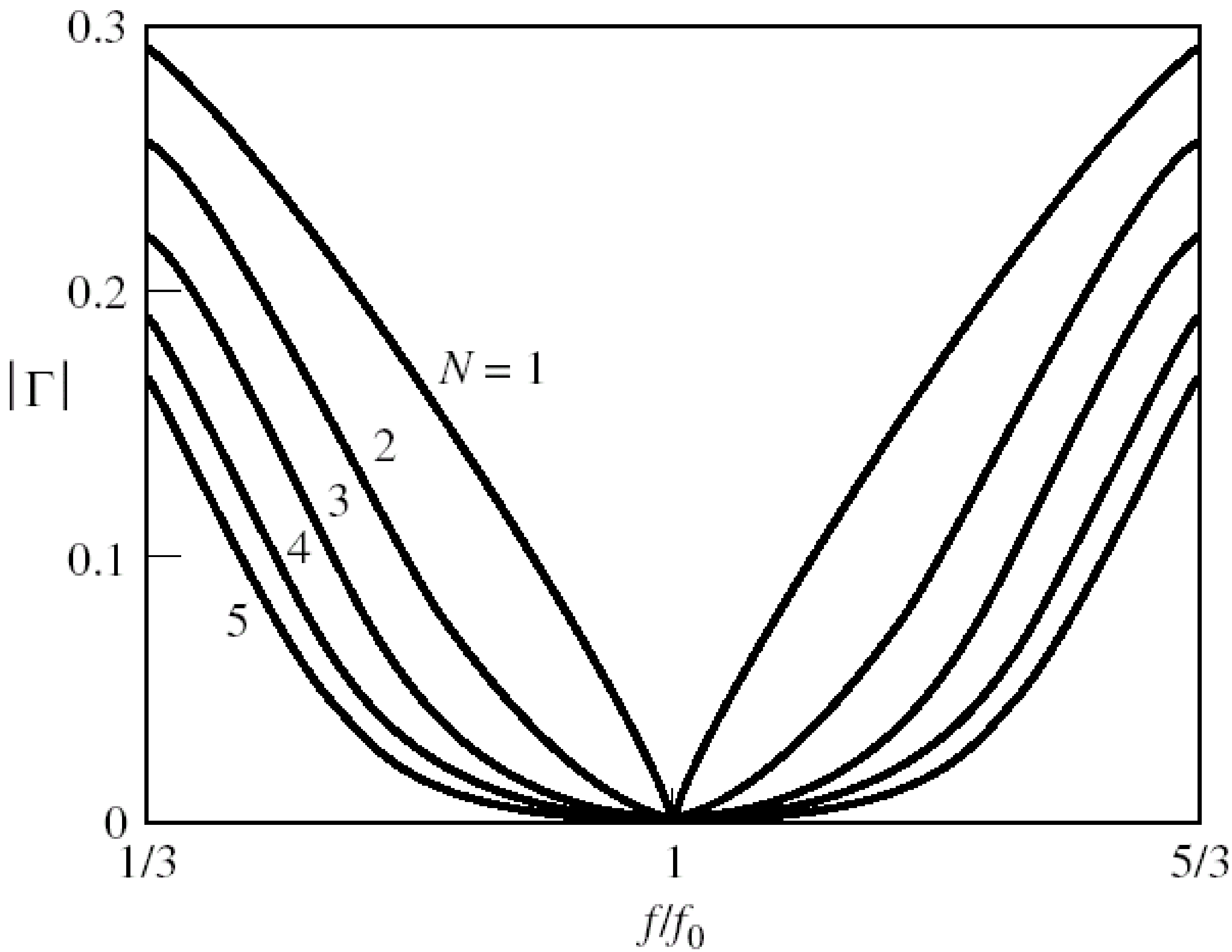
$$\Gamma_m = |\Gamma(\theta_m)| = 2^N \cdot |A| \cdot |\cos\theta_m|^N$$



$$\theta_m = \cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{\frac{1}{N}}\right]$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi}\cdot\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{\frac{1}{N}}\right]$$

Bandwidth



Binomial multisection transformer
Exact results

Z_L/Z_0	$N=2$		$N=3$			$N=4$			
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	$N=5$					$N=6$					
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

Example

- Design a three-section binomial transformer to match a 30Ω load to a 100 Ω line at f_o=3GHz, Γ_m=0.1
  - N = 3

$$Z_L = 30\Omega \qquad Z_0 = 100\,\Omega$$

$$A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}}\ln\frac{Z_L}{Z_0} = -0.07525$$

$$C_3^0 = \frac{3!}{3!\cdot 0!} = 1 \qquad C_3^1 = \frac{3!}{2!\cdot 1!} = 3 \qquad C_3^2 = \frac{3!}{1!\cdot 2!} = 3$$

Example

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_3^0 \ln \frac{Z_L}{Z_0} = \ln 100 + 2^{-3} \cdot 1 \cdot \ln \frac{30}{100} = 4.455$$

$$Z_1 = 86.03\Omega$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_3^1 \ln \frac{Z_L}{Z_0} = \ln 86.03 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 4.003$$

$$Z_2 = 54.77\Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_3^2 \ln \frac{Z_L}{Z_0} = \ln 54.77 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 3.552$$

$$Z_3 = 34.87\Omega$$

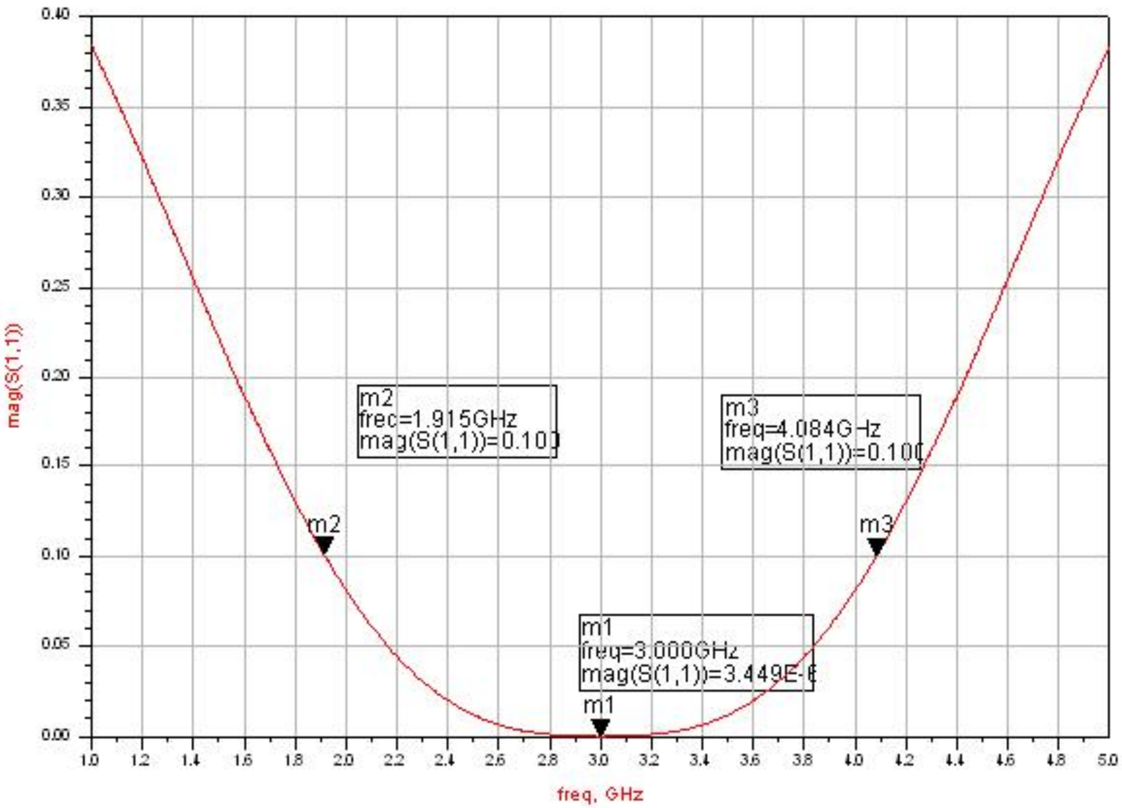
Example

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right] = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{0.1}{0.07525}\right)^{1/3}\right] = 0.74$$

$$\Delta f = 2.22GHz$$

Simulation

- Similarly Lab. 1



$$\Delta f = 2.169 GHz$$

$$\left|\Gamma(3GHz)\right| = 3.5 \cdot 10^{-6}$$

Chebyshev multisection transformer

- The response of this multisection impedance transformer is **equal-ripple** in passband
- optimizes (increases) bandwidth at the expense of passband ripple
- We match the $\Gamma(\theta)$ function with an desired Chebyshev polynomial

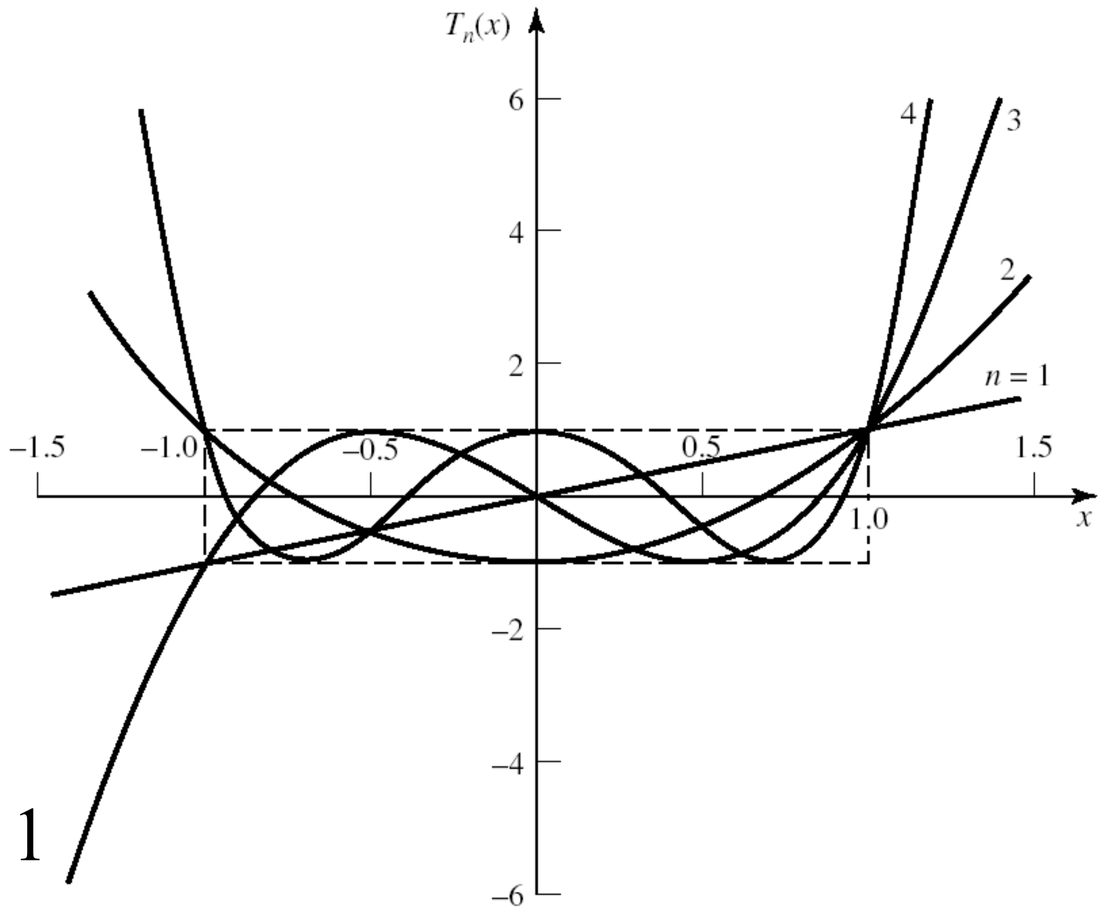
Chebyshev polynomials

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



- equal-ripple

$$-1 \le x \le 1 \quad \Rightarrow \quad |T_n(x)| \le 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

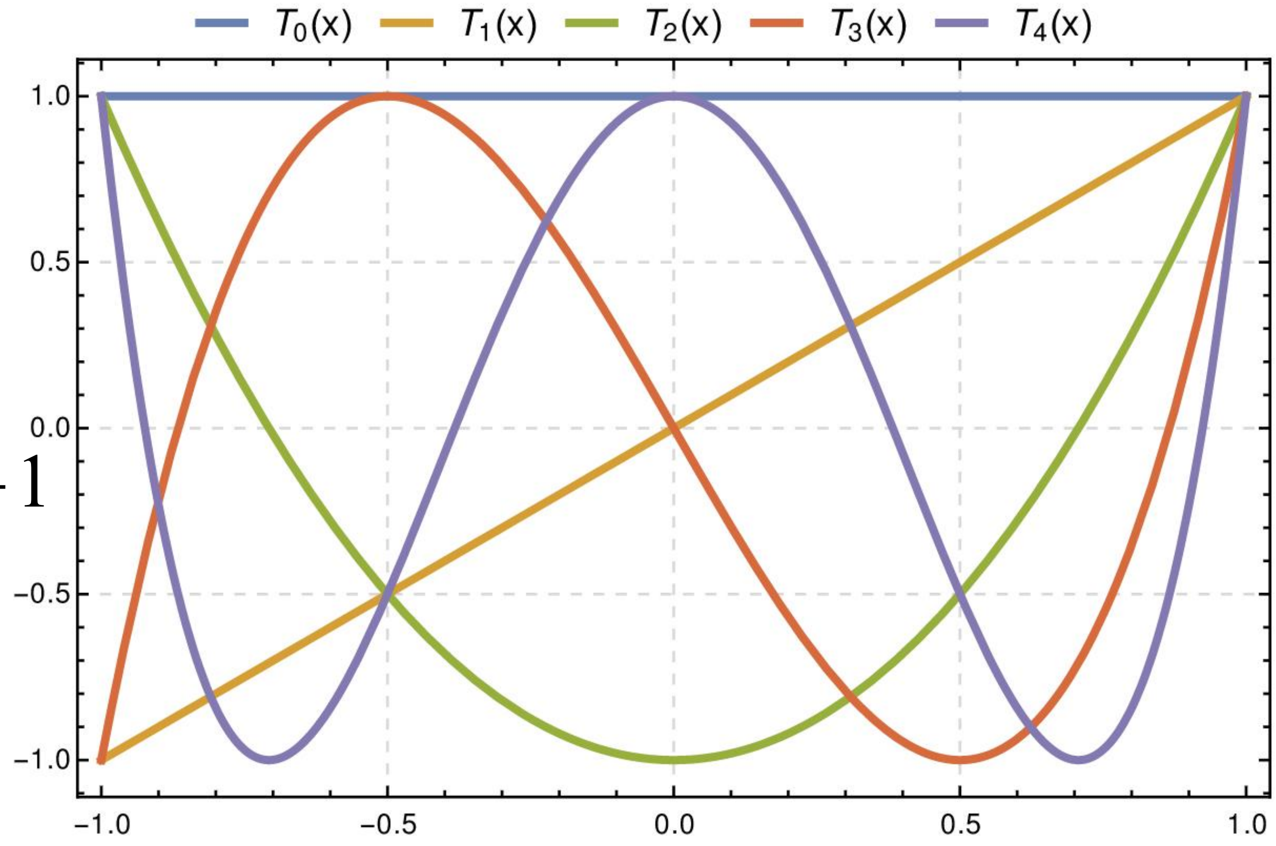
Chebyshev polynomials

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



- **equal-ripple**

$$-1 \leq x \leq 1 \quad \Rightarrow \quad |T_n(x)| \leq 1$$

Chebyshev polynomials

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

last item:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad N \text{ even}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad N \text{ odd}$$

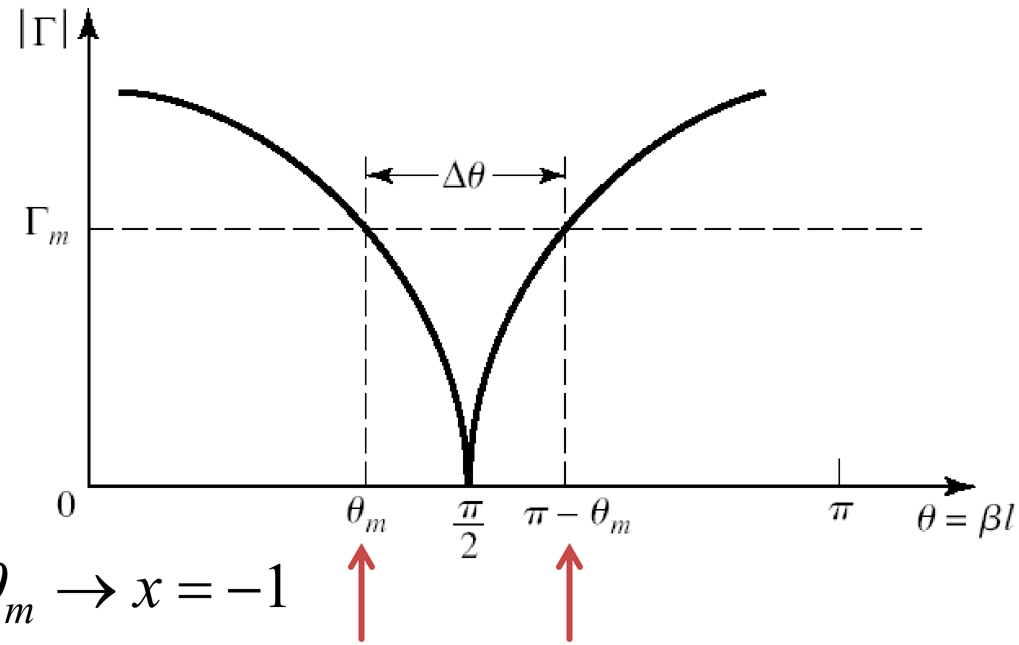
$$x = \cos\theta \qquad |x| < 1$$

- We can show that: $T_n(\cos\theta) = \cos(n\theta)$

$$T_n(x) = \cos(n \arccos(x)) \quad |x| < 1 \qquad T_n(x) = \cosh\left(n \cosh^{-1}(x)\right) \quad |x| > 1$$

Chebyshev multisection transformer

- variable change so we map:
  - bandwidth -> [-1,1]



$$\theta = \theta_m \rightarrow x = 1$$

$$\theta = \pi - \theta_m \rightarrow x = -1$$

$$x \equiv \frac{\cos\theta}{\cos\theta_m}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$x = \sec\theta_m \cos\theta$$

Chebyshev multisection transformer

- We search the coefficients to obtain a Chebyshev polynomial

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \left(e^{jN\theta} + e^{-jN\theta}\right) + \Gamma_1 \cdot \left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + \Gamma_2 \cdot \left(e^{j(N-4)\theta} + e^{-j(N-4)\theta}\right) + \cdots\right]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

last item:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ even}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad n \text{ odd}$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N\left(\sec\theta_m \cos\theta\right)$$

Chebyshev polynomials

$$(\cos\theta)^k \Leftrightarrow \cos k\theta$$

$$T_1(x) = x \qquad T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(x) = 2x^2 - 1 \qquad T_2(\sec\theta_m \cos\theta) = 2\sec^2\theta_m \cos^2\theta - 1$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m (1 + \cos 2\theta) - 1$$

$$T_3(x) = 4x^3 - 3x \qquad T_3(\sec\theta_m \cos\theta) = 4\sec^3\theta_m \cos^3\theta - 3\sec\theta_m \cos\theta$$

$$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$

$$\cos 3\theta = (2\cos^2\theta - 1)\cos\theta - 2(1 - \cos^2\theta)\cos\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

Chebyshev multisection transformer

$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m (1 + \cos 2\theta) - 1$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

$$T_4(\sec\theta_m \cos\theta) = \sec^4\theta_m (\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m (\cos 2\theta + 1) + 1$$

- We search coefficients of Γ(θ) function to obtain a Chebyshev polynomial

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots]$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec\theta_m \cos\theta)$$

last item:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad N \text{ even}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad N \text{ odd}$$

Chebyshev multisection transformer

- A, θ →0 , 0 length sections, the sections disappear

$$\Gamma(0)=\frac{Z_L-Z_0}{Z_L+Z_0}=A\cdot T_N(\sec\theta_m) \qquad A=\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\frac{1}{T_N(\sec\theta_m)} \qquad \boxed{\Gamma_m=|A|}$$

$$T_N(\sec\theta_m)=\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\cong\frac{1}{2\Gamma_m}\left|\ln\frac{Z_L}{Z_0}\right|$$

$$T_n(x)=\cosh\left(n\cosh^{-1}(x)\right)$$

$$\sec\theta_m=\cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\right)\right]\cong\cosh\left[\frac{1}{N}\cosh^{-1}\left(\left|\frac{\ln(Z_L/Z_0)}{2\Gamma_m}\right|\right)\right]$$

- we compute θ_m for maximum acceptable value Γ_m (ripple) then bandwidth is:

$$\frac{\Delta f}{f_0}=\frac{2(f_0-f_m)}{f_0}=2-\frac{4\theta_m}{\pi}$$

Chebyshev multisection transformer

- Design procedure, approximate solutions

$$\Gamma_m = |A|$$

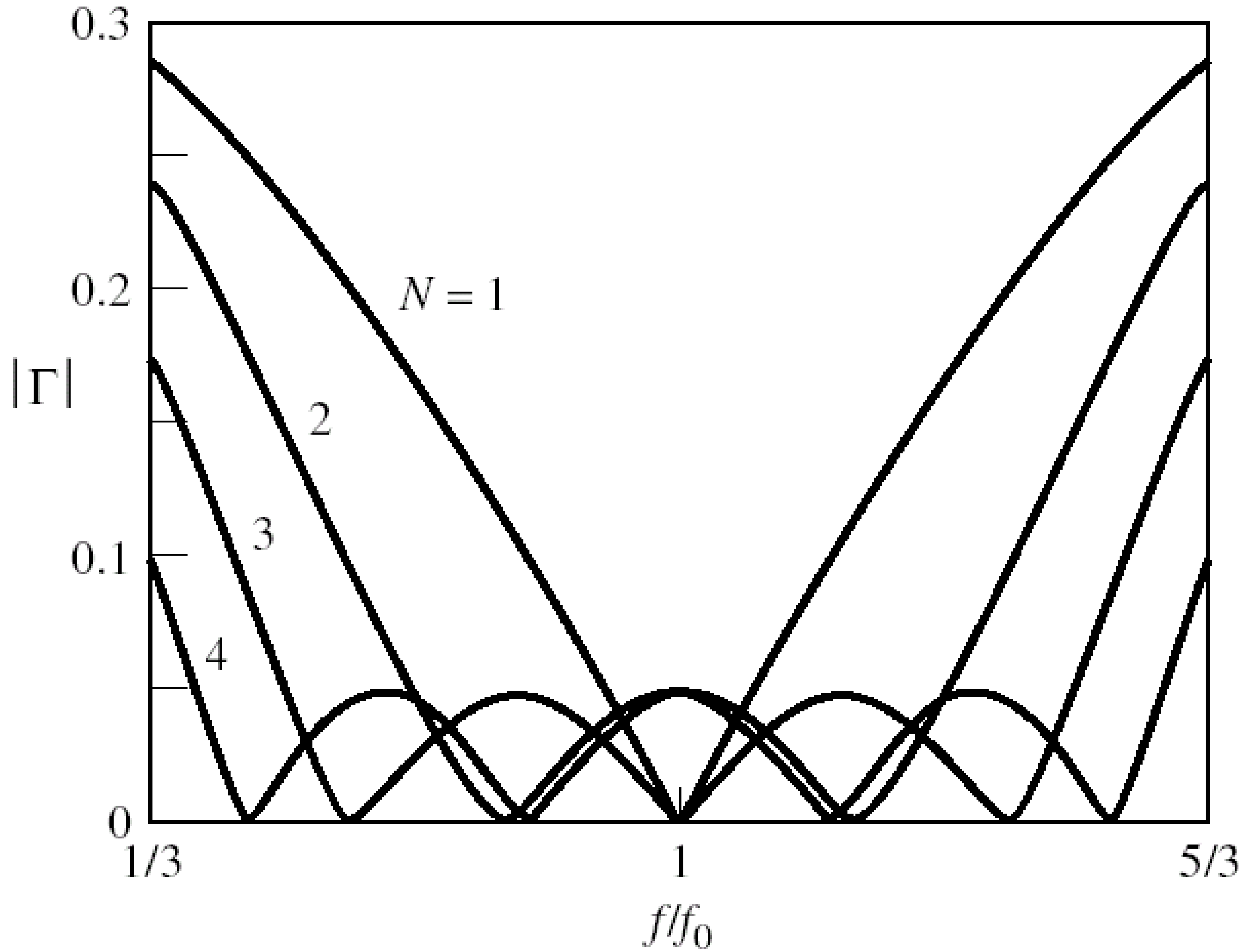
$$A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec\theta_m)}$$

- Sign of A depends on Z_L <> Z_o
- Compute sec θ_m
- Write down the Chebyshev polynomial for the order of your choice and identify cos kθ coefficients

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots]$$

$$\ln\frac{Z_{n+1}}{Z_n} \cong 2\cdot\Gamma_n \qquad \ln Z_{n+1} \cong \ln Z_n + 2\cdot\Gamma_n$$

Bandwidth



Chebyshev multisection transformer Exact results

	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

$N = 4$

	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

Example

- Design a three-section Chebyshev transformer to match a 30Ω load to a 100 Ω line at f_o=3GHz, Γ_m=0.1
  - N = 3 $\quad Z_L = 30\Omega \quad Z_0 = 100\,\Omega$

$$\Gamma(\theta) = 2e^{-j3\theta}\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = Ae^{-j3\theta} T_3\left(\sec\theta_m \cos\theta\right)$$

$$|A| = \Gamma_m = 0.1 \qquad A = \frac{Z_L - Z_0}{Z_L + Z_0}\cdot\frac{1}{T_N\left(\sec\theta_m\right)} \qquad Z_L < Z_0 \rightarrow A < 0 \qquad A = -0.1$$

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cdot\cosh^{-1}\left(\left|\frac{\ln Z_L/Z_0}{2\Gamma_m}\right|\right)\right] = \cosh\left[\frac{1}{3}\cdot\cosh^{-1}\left(\left|\frac{\ln(30/100)}{2\cdot 0.1}\right|\right)\right] = 1.362$$

$$\theta_m = \arccos\left(\frac{1}{\sec\theta_m}\right) = 0.746\,rad = 42.76^\circ$$

Example

$$2\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = A\sec^3\theta_m\left(\cos 3\theta + 3\cos\theta\right) - 3A\sec\theta_m \cos\theta$$

$\cos 3\theta$ $\quad 2\Gamma_0 = A\sec^3\theta_m$ $\quad \Gamma_0 = -0.1263$

$\cos\theta$ $\quad 2\Gamma_1 = 3A\left(\sec^3\theta_m - \sec\theta_m\right)$ $\quad \Gamma_1 = -0.1747$

symmetry: $\Gamma_3 = \Gamma_0; \quad \Gamma_2 = \Gamma_1$

Example

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2 \cdot \Gamma_0 = \ln 100 - 2 \cdot 0.1263 = 4.353$$

$$\Gamma_0 = -0.1263$$

$$Z_1 = 77.68\Omega$$

$$\Gamma_1 = -0.1747$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2 \cdot \Gamma_1 = \ln 77.68 - 2 \cdot 0.1747 = 4.003$$

$$Z_2 = 54.77\Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2 \cdot \Gamma_2 = \ln 54.77 - 2 \cdot 0.1747 = 3.654$$

$$Z_3 = 38.62\Omega$$

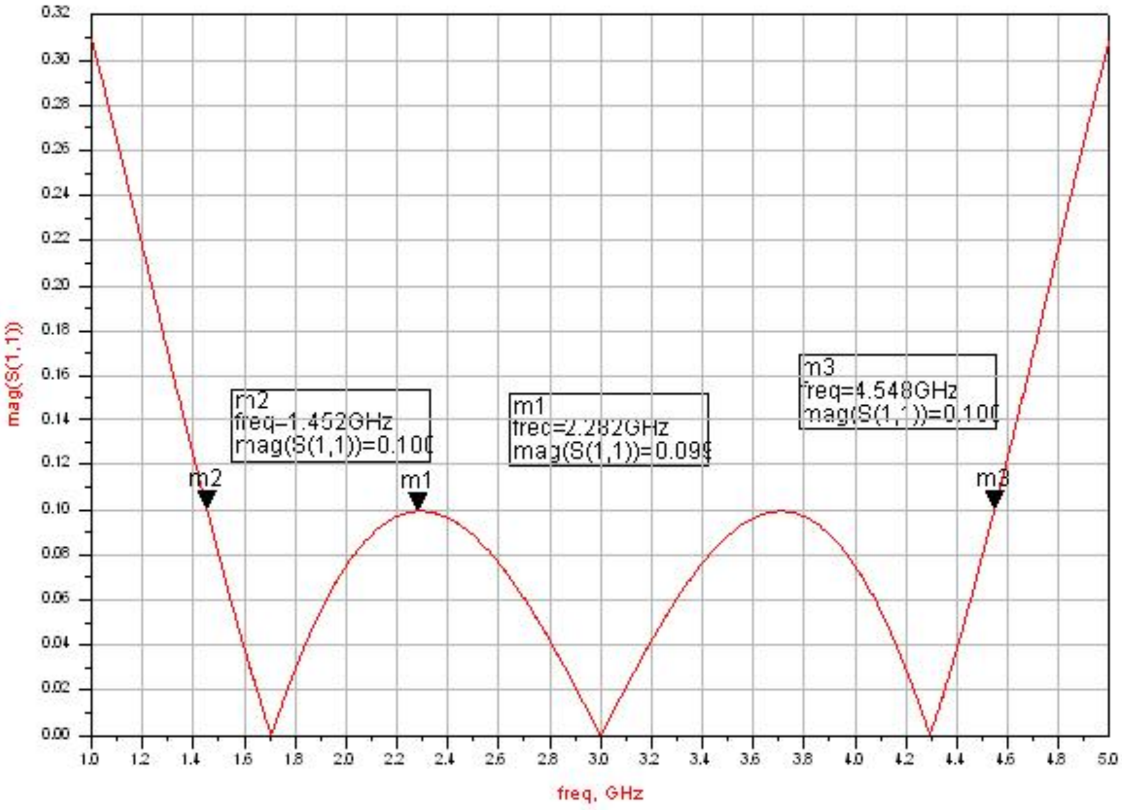
Example

$$\frac{\Delta f}{f_0} = \frac{2\left(f_0 - f_m\right)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4 \cdot 42.76°}{180°} = 1.045$$

$$\Delta f = 3.15 GHz$$

Simultion

- Similarly Lab. 1



$$\Delta f = 3.096GHz$$

$$\left|\Gamma(3GHz)\right| = 4.17 \cdot 10^{-5}$$

$$\left|\Gamma(2.282GHz)\right| = 0.09925$$

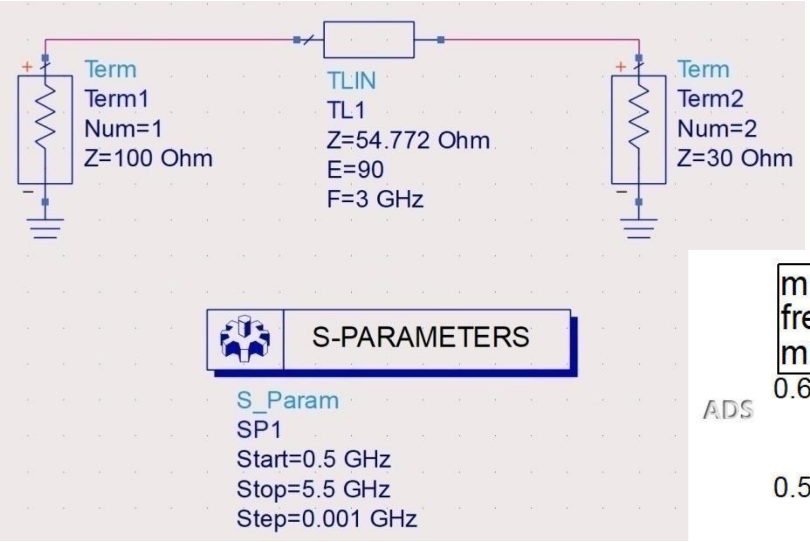
Exact solutions

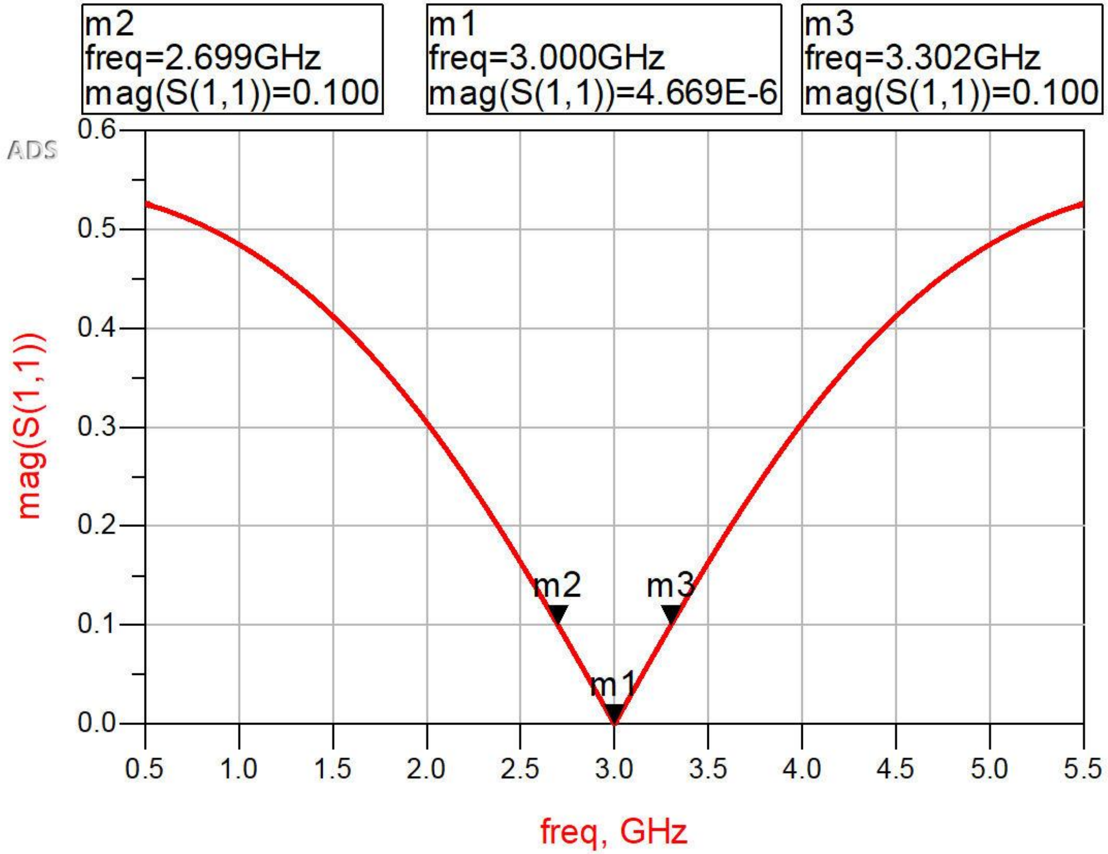
- G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks,and Coupling Structures, Artech House Books, Dedham, Mass. 1980*

Laboratory 1

Impedance Matching

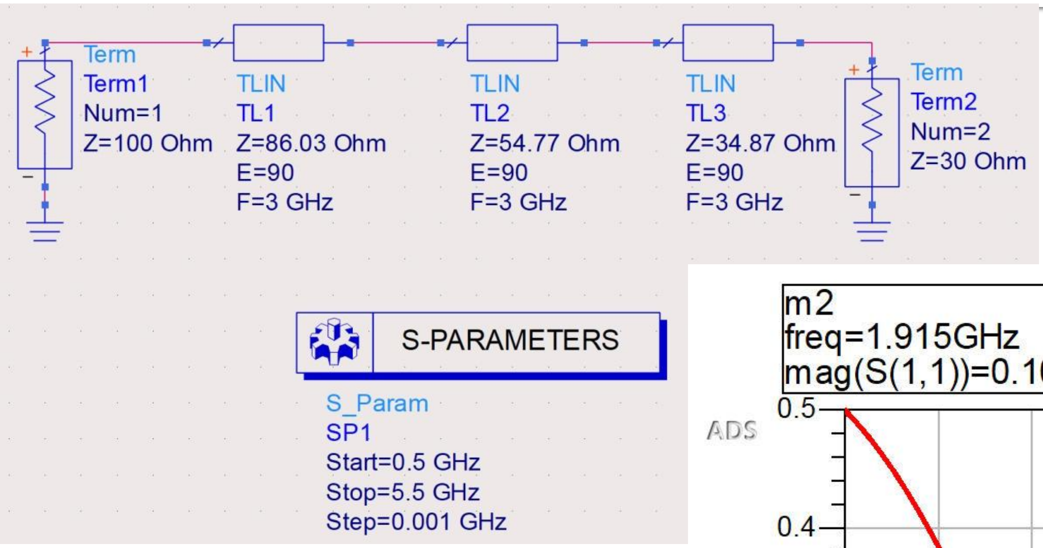
The quarter-wave transformer

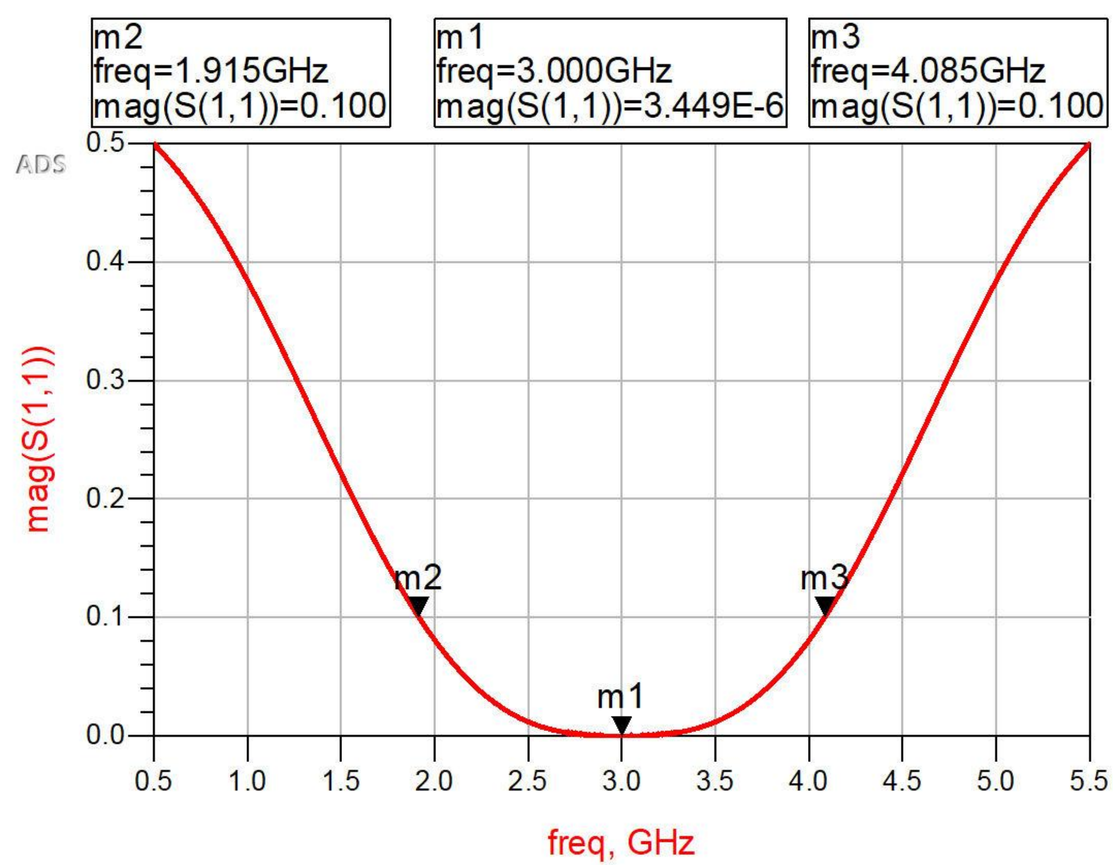




$$\Delta f = 0.603GHz$$

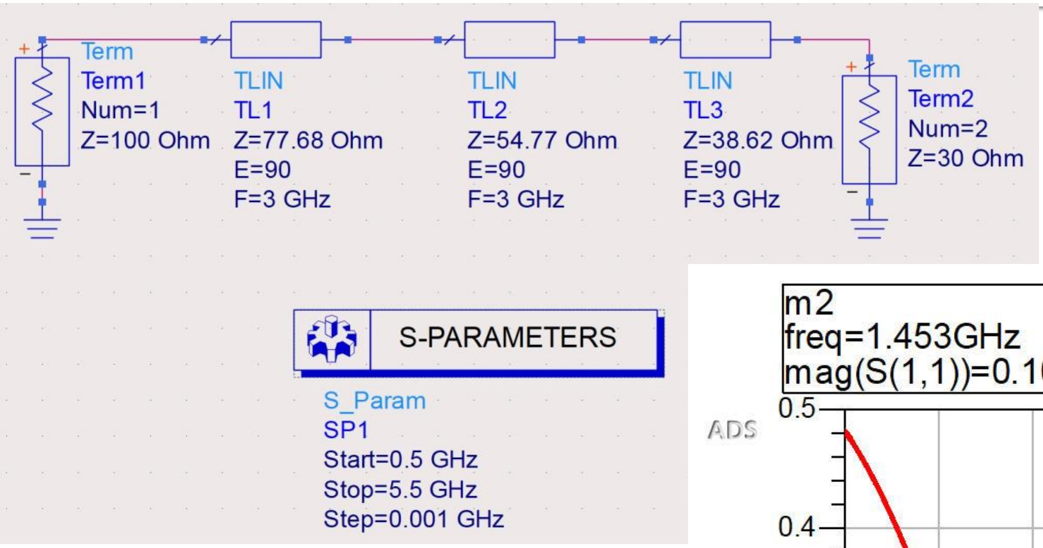
Binomial multisection transformer

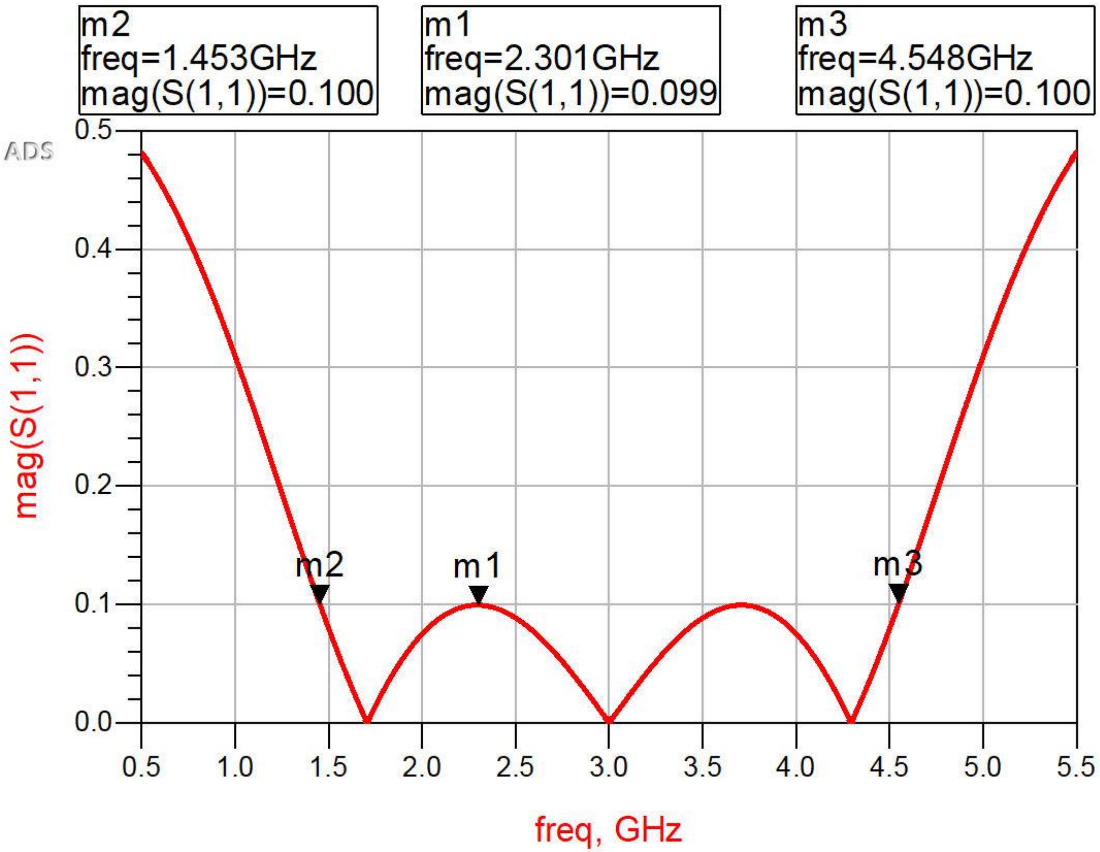




$$\Delta f = 2.169GHz$$

Chebyshev multisection transformer





$$\Delta f = 3.096GHz$$

General theory

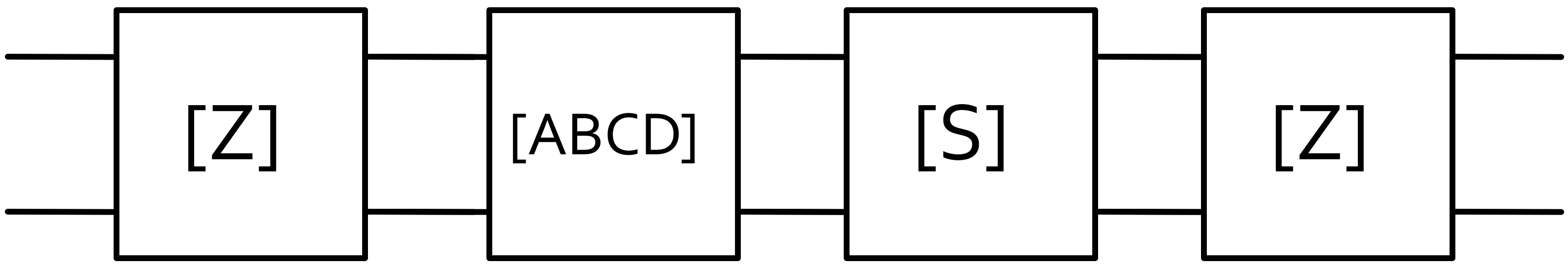
Microwave Network Analysis

Course Topics

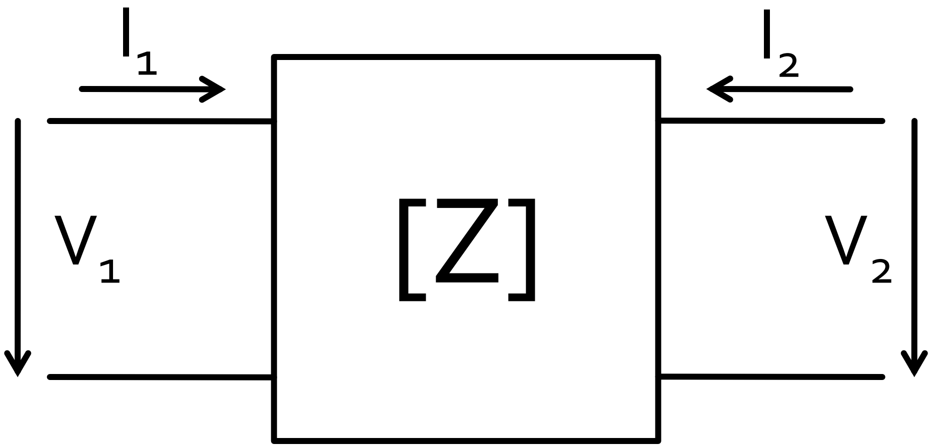
- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Network Analysis

- We try to separate a complex circuit into individual blocks
- These are analyzed separately (decoupled from the rest of the circuit) and are characterized only by the port level signals (**black box**)
- Network-level analysis allows you to put together individual block results and get a total result for the entire circuit



Impedance matrix – Z



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

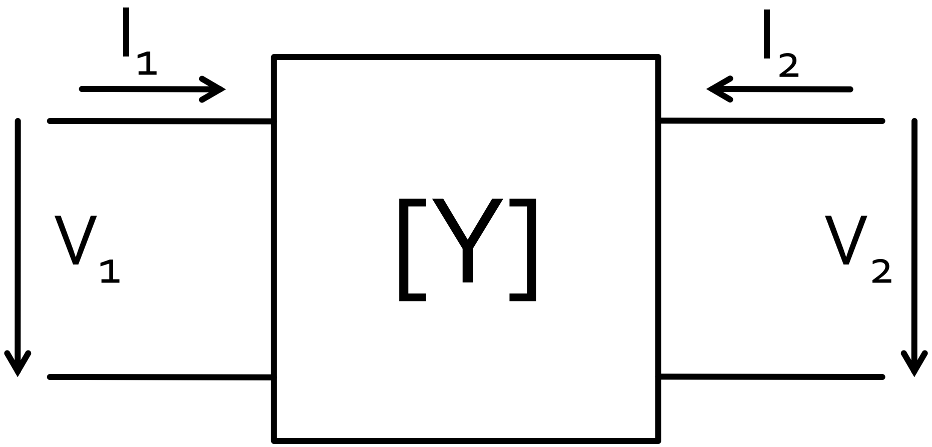
$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$V_1 = Z_{11} \cdot I_1 \big|_{I_2=0} \qquad Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

- Z11 – input impedance with open-circuited output

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} \qquad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

Admittance matrix – Y



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

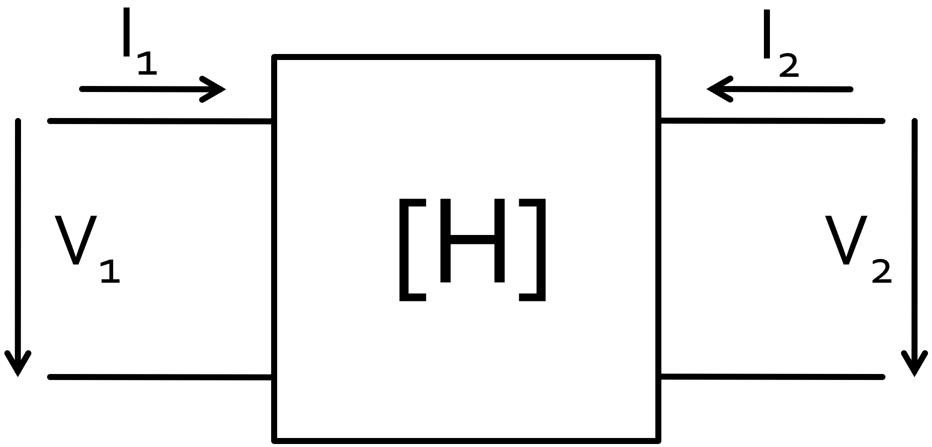
$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2$$
$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2$$

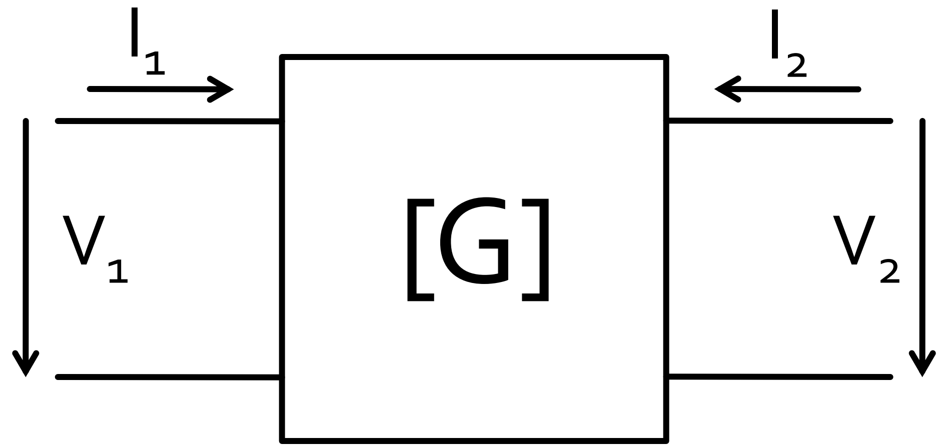
$$I_1 = Y_{11} \cdot V_1 \big|_{V_2=0} \qquad Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$

- Y11 – input admittance with short-circuited output

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \qquad Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \qquad Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} \qquad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

Hybrid matrices – H and G





$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$H_{21} = \left. \frac{I_2}{I_1} \right|_{V_2 = 0 \, or \, H_{22} \to \infty}$$

- h_{21E} widely used for Bipolar Transistors, common emitter topology (or β, h_{22E} >>)

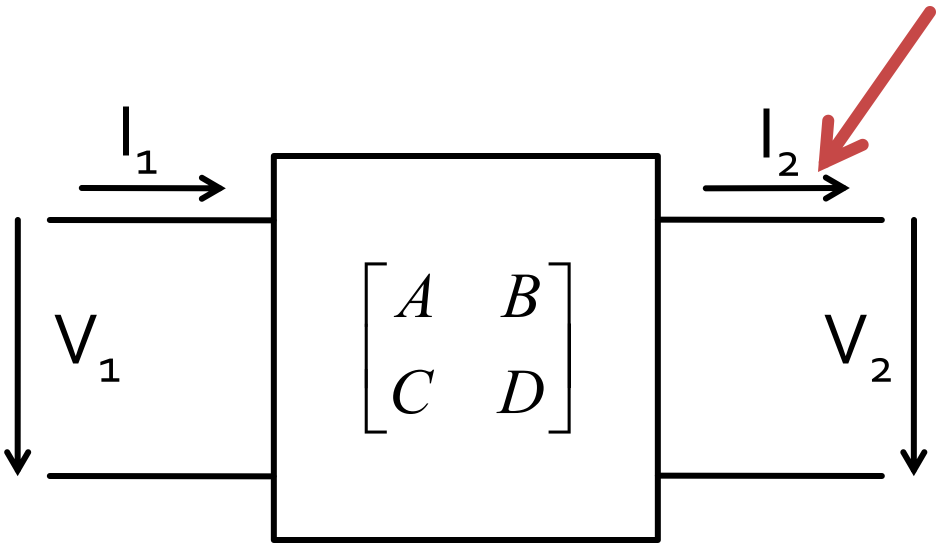
Network Analysis

- Each matrix is best suited for a particular mode of port excitation (V, I)
  - matrix H in common emitter connection for TB: I_B, V_{CE}
  - matrices provide the associated quantities depending on the "attack" ones
- Traditional notation of Z, Y, G, H parameters is in lowercase (z, y, g, h)
- In microwave analysis we prefer the notation in uppercase to avoid confusion with the **normalized parameters**

$$z = \frac{Z}{Z_0} \qquad y = \frac{Y}{Y_0} = \frac{1/Z}{1/Z_0} = \frac{Z_0}{Z} = Z_0 \cdot Y$$

$$z_{11} = \frac{Z_{11}}{Z_0} \qquad y_{11} = \frac{Y_{11}}{Y_0} = Z_0 \cdot Y_{11}$$

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

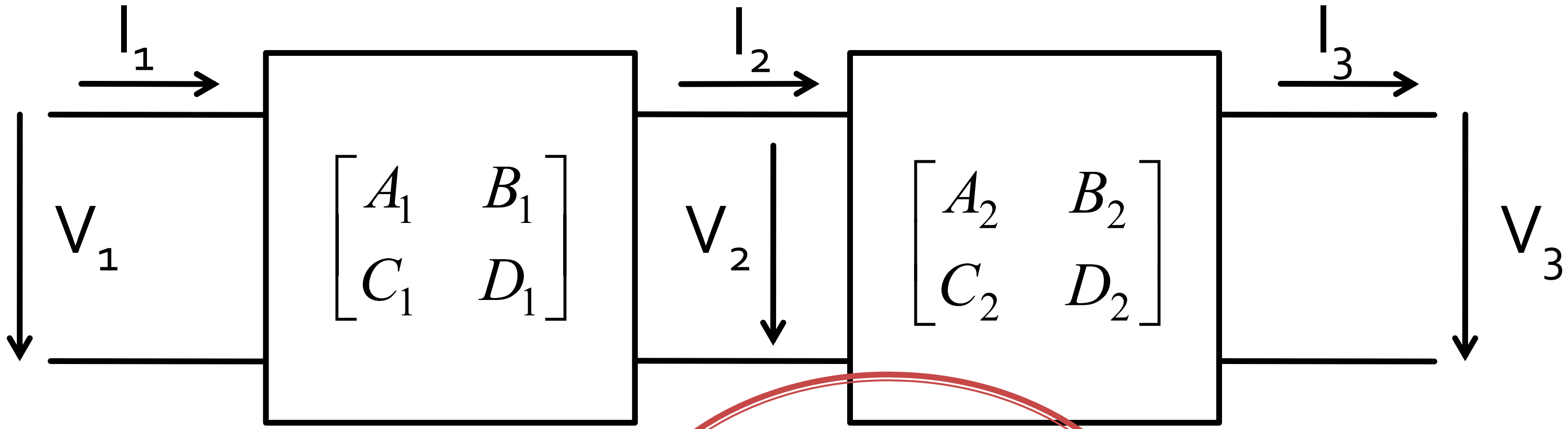
$$V_1 = A \cdot V_2 + B \cdot I_2$$
$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \qquad B = \left. \frac{V_1}{I_2} \right|_{V_2=0} \qquad C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \qquad D = \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

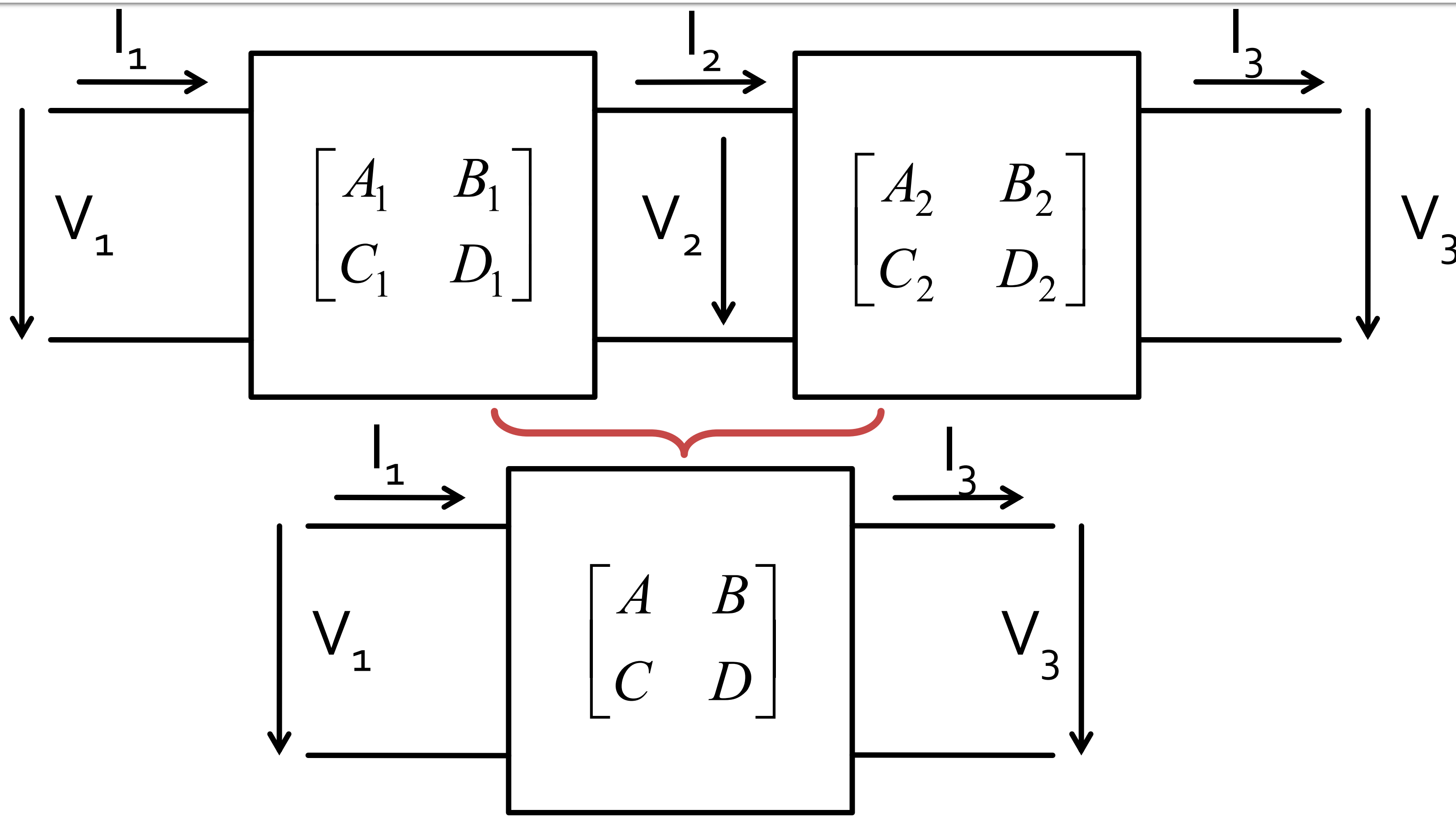
ABCD (transmission) matrix

- This 2X2 matrix characterizes the "input"/"output" relation
- Allows easy chaining of multiple two-ports



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

ABCD (transmission) matrix



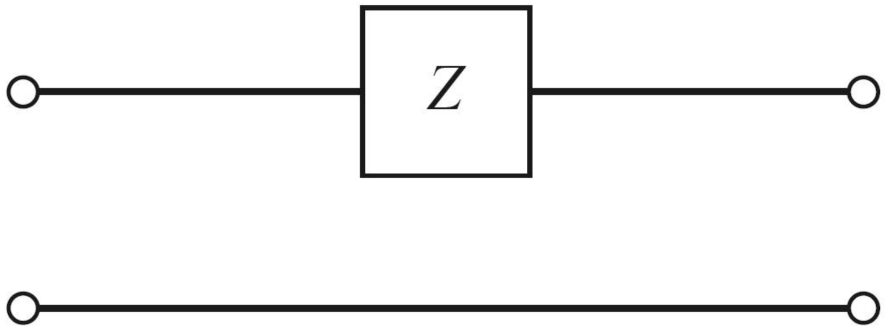
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

ABCD (transmission) matrix

- suitable **only** for two-port networks (Z, Y can be easily extended for multiport / n-ports)
- allows easy coupling of multiple elements
- allows the calculation of complex circuits with one input and one output by breaking them in individual component blocks
- a library of ABCD matrices for elementary two-port networks can be built up

Library of ABCD matrices

- Series impedance



$$A = 1 \qquad B = Z$$
$$C = 0 \qquad D = 1$$

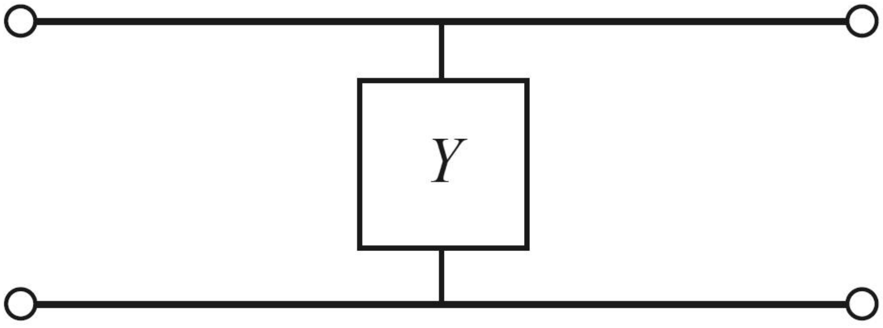
$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0 \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{I_1} = 1$$

Library of ABCD matrices

- Shunt admittance



$$A = 1 \qquad B = 0$$

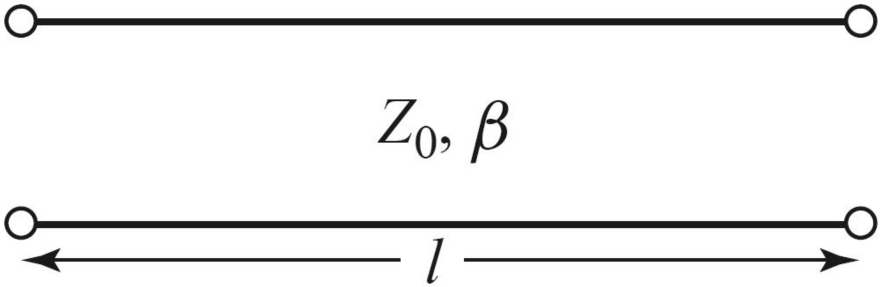
$$C = Y \qquad D = 1$$

$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Homework!

Library of ABCD matrices

- Transmission line



$$A = \cos \beta \cdot l$$

$$B = j \cdot Z_0 \cdot \sin \beta \cdot l$$

$$C = j \cdot Y_0 \cdot \sin \beta \cdot l$$

$$D = \cos \beta \cdot l$$

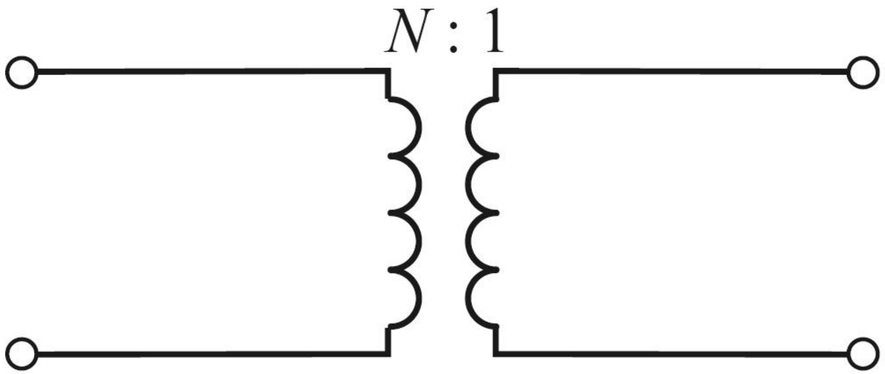
Homework!

$$\begin{bmatrix} \cos \beta \cdot l & j \cdot Z_0 \cdot \sin \beta \cdot l \\ j \cdot Y_0 \cdot \sin \beta \cdot l & \cos \beta \cdot l \end{bmatrix}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Library of ABCD matrices

- Transformer



$$A = N \qquad B = 0$$

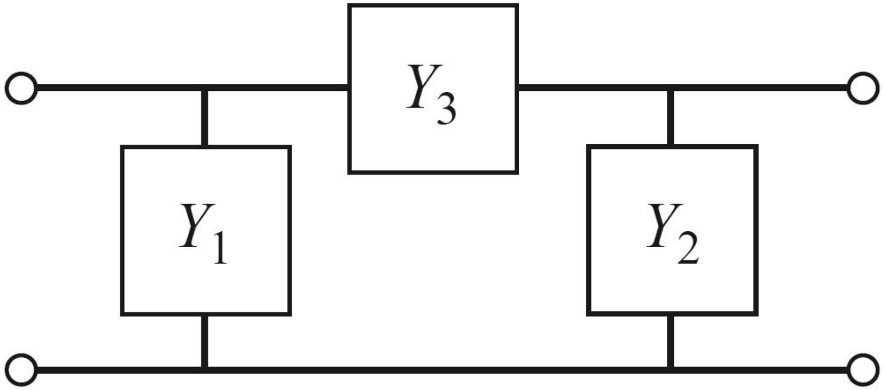
$$C = 0 \qquad D = \frac{1}{N}$$

$$\begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix}$$

Homework!

Library of ABCD matrices

- π network



$$A = 1 + \frac{Y_2}{Y_3}$$

$$B = \frac{1}{Y_3}$$

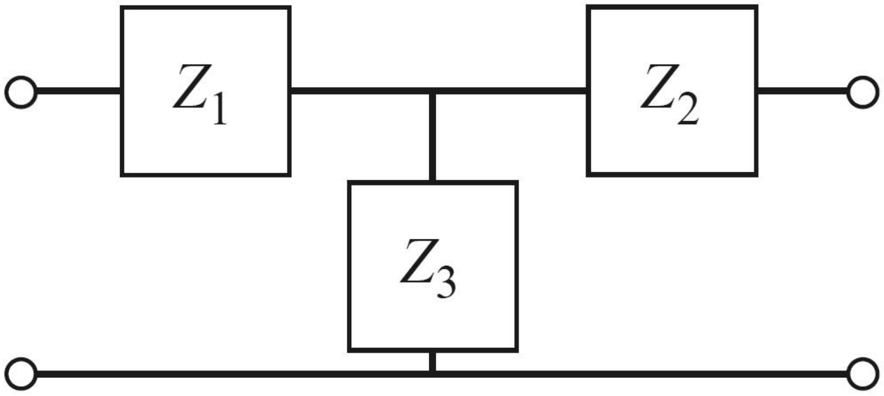
$$C = Y_1 + Y_2 + \frac{Y_1 \cdot Y_2}{Y_3}$$

$$D = 1 + \frac{Y_1}{Y_3}$$

Homework!

Library of ABCD matrices

- T network



$$A = 1 + \frac{Z_1}{Z_3}$$

$$B = Z_1 + Z_2 + \frac{Z_1 \cdot Z_2}{Z_3}$$

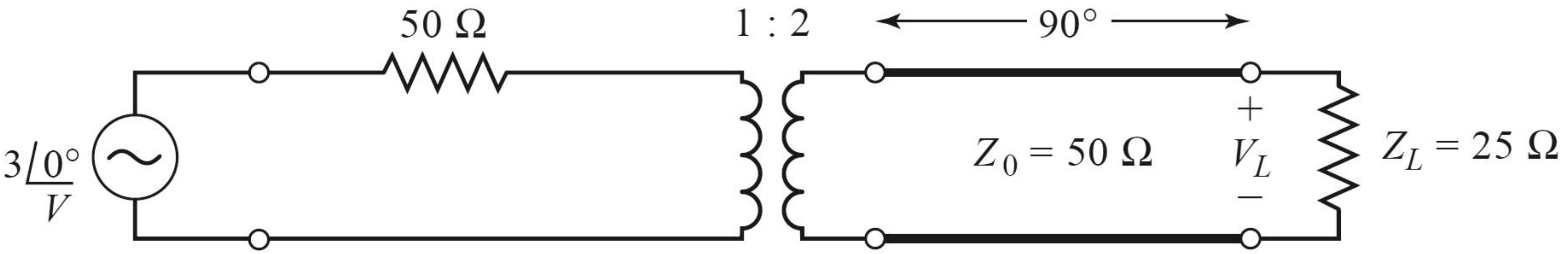
$$C = \frac{1}{Z_3}$$

$$D = 1 + \frac{Z_2}{Z_3}$$

Homework!

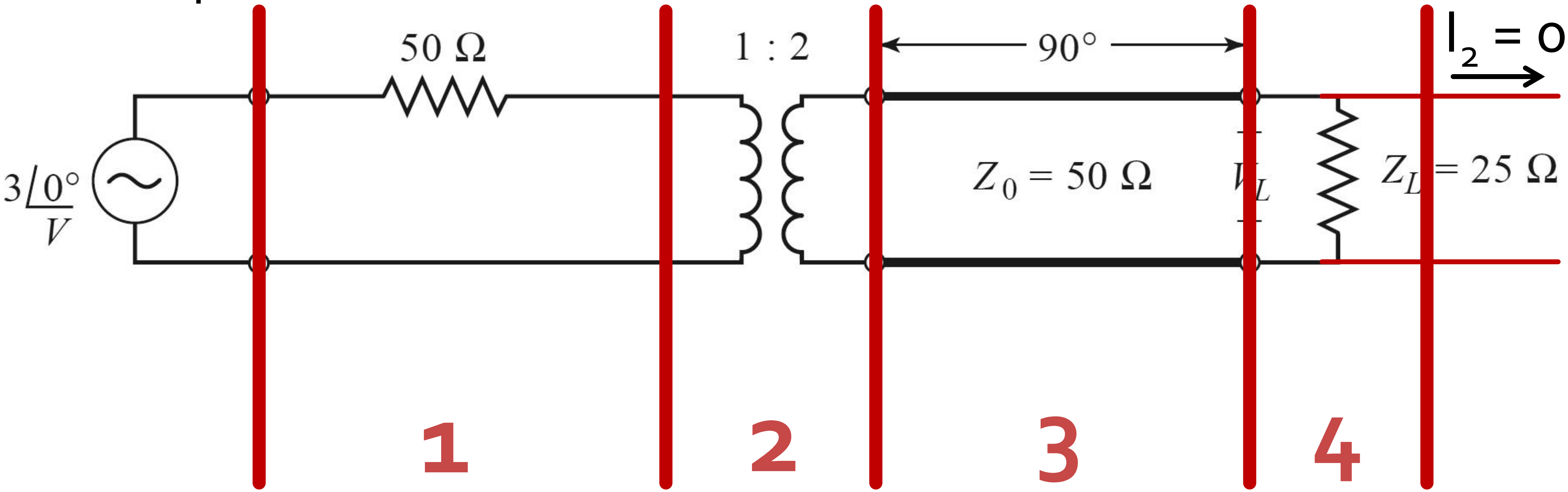
Example for ABCD matrix

- Find the voltage V_L across the load resistor in the circuit shown below (Pozar/exam problem)



Example for ABCD matrix

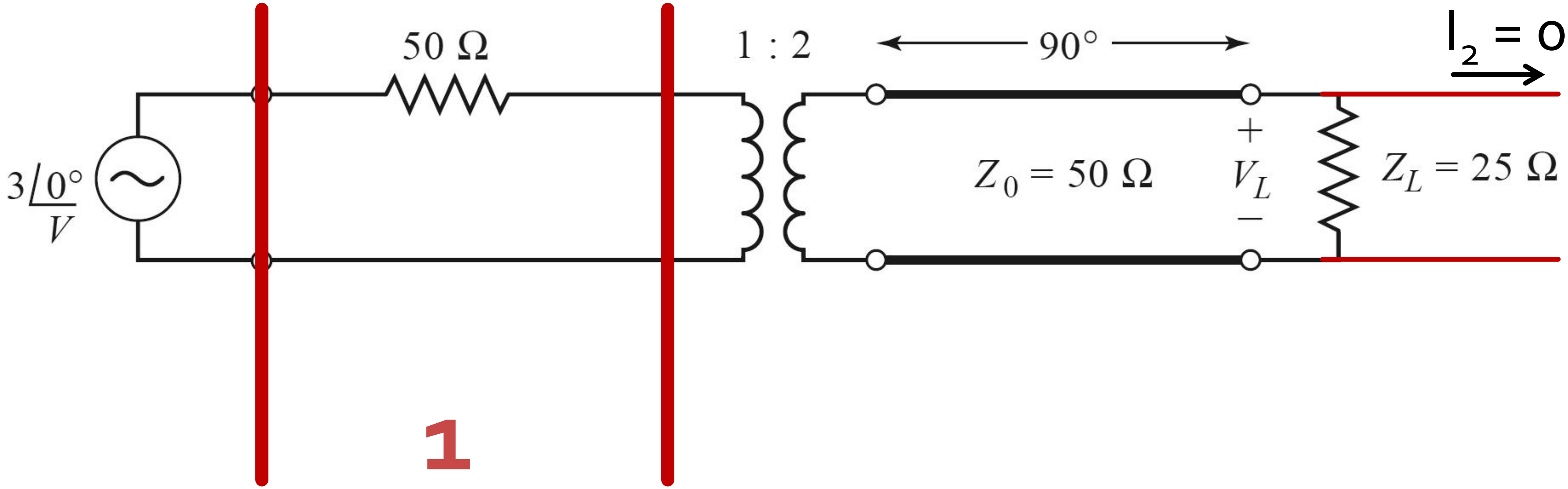
- We break the circuit in elementary sections
- Sources are left outside
- If necessary, input and output ports are created (and left open-circuited)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M_1 \cdot M_2 \cdot M_3 \cdot M_4 \quad V_1 = A \cdot V_2 + B \cdot I_2 \Big|_{I_2=0} \quad V = A \cdot V_L \rightarrow V_L = \frac{V}{A}$$

Example for ABCD matrix

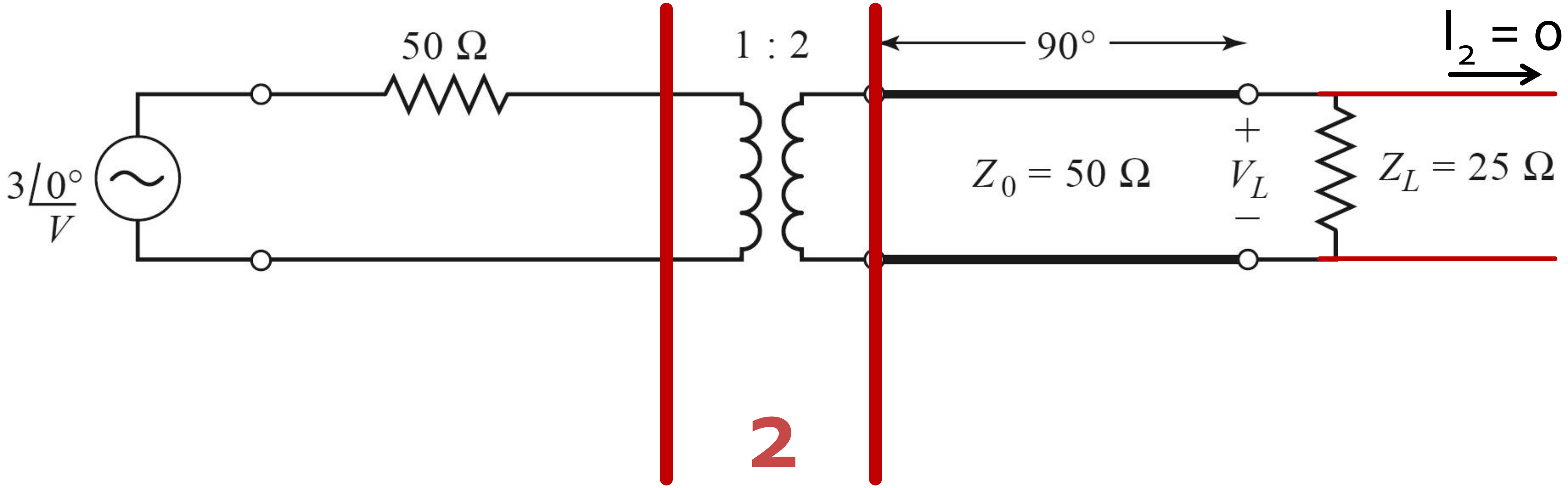
- M_1 , series impedance



$$M_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$$

Example for ABCD matrix

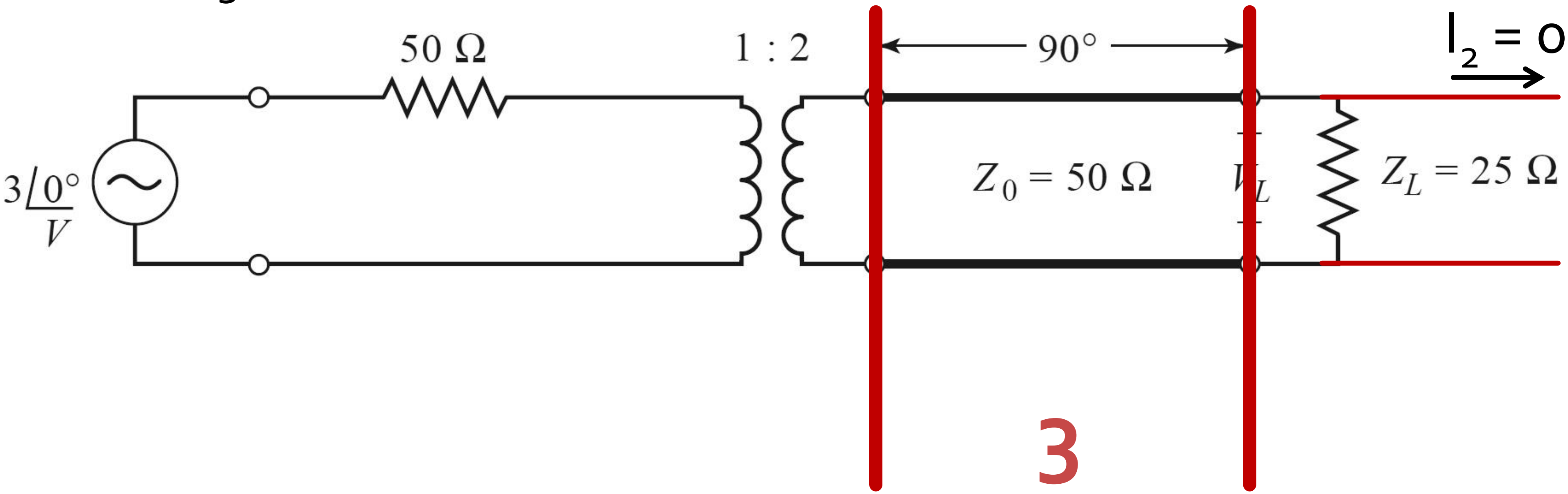
- M_2 , 1:2 transformer



$$M_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

Example for ABCD matrix

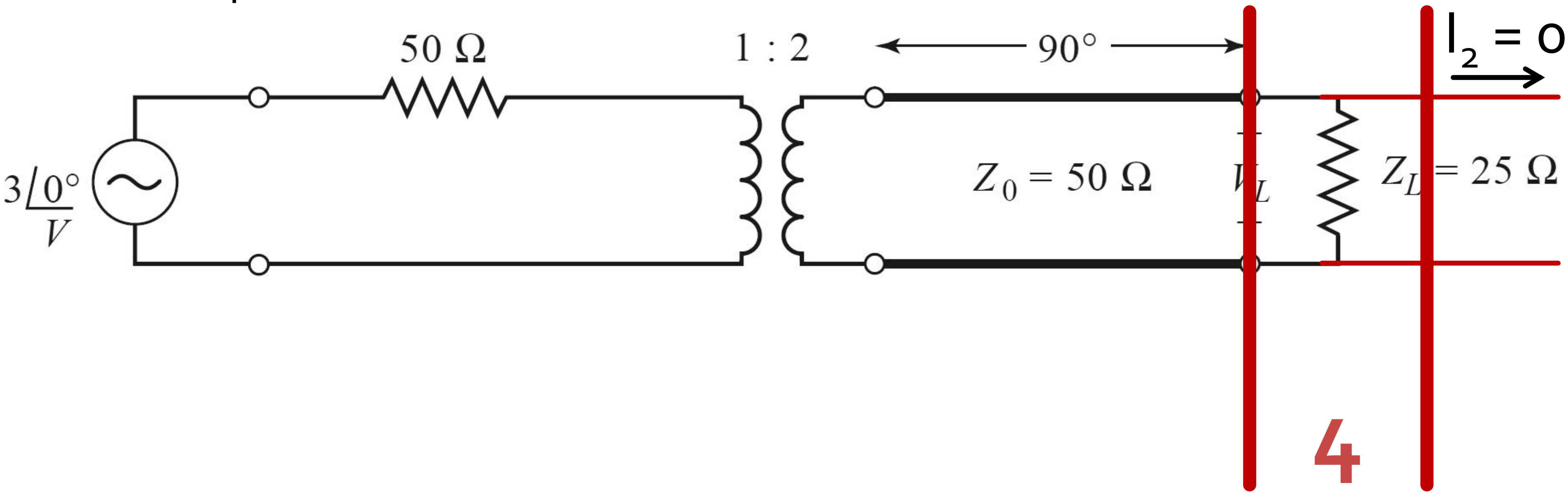
- M_3 , series transmission line, E = 90°



$$M_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & 50 \cdot j \\ \frac{j}{50} & 0 \end{bmatrix}$$

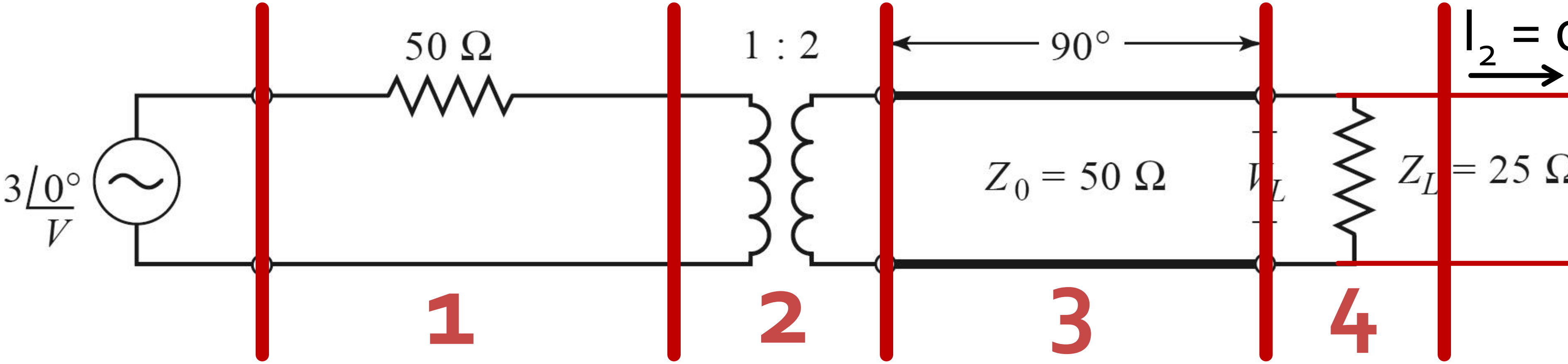
Example for ABCD matrix

- M_4 , shunt impedance/admittance



$$M_4 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix}$$

Example for ABCD matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 50 \cdot j \\ \frac{j}{50} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot j & 25 \cdot j \\ \frac{j}{25} & 0 \end{bmatrix}$$

$$V_L = \frac{V}{A} = \frac{3\angle 0^\circ}{3 \cdot j} = 1\angle -90^\circ$$

(Somewhat!) Specific theory

Microwave Network Analysis

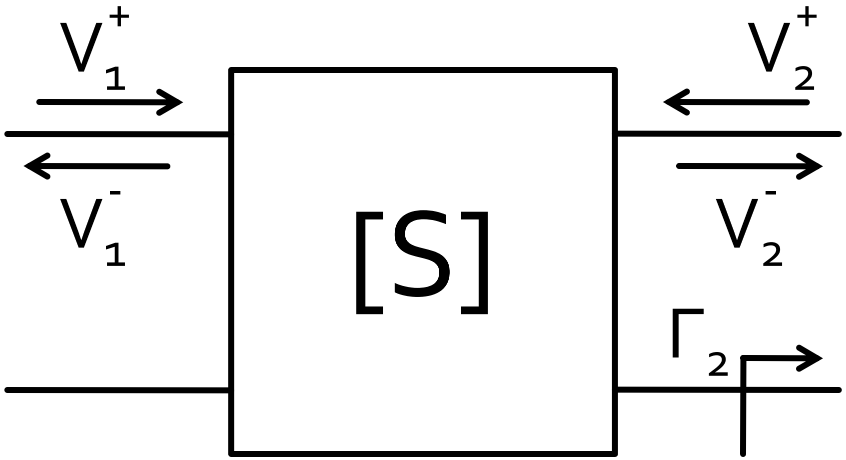
The lossless line

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

- Average power flow is constant along the line
  - (**no** P_{avg}(**z**))
  - can be measured
- We can use the power to characterize the amplitude of a signal
  - a very "energetic" (basic physics) point of view
  - more power = "more" signal

Scattering matrix – S

- Scattering parameters



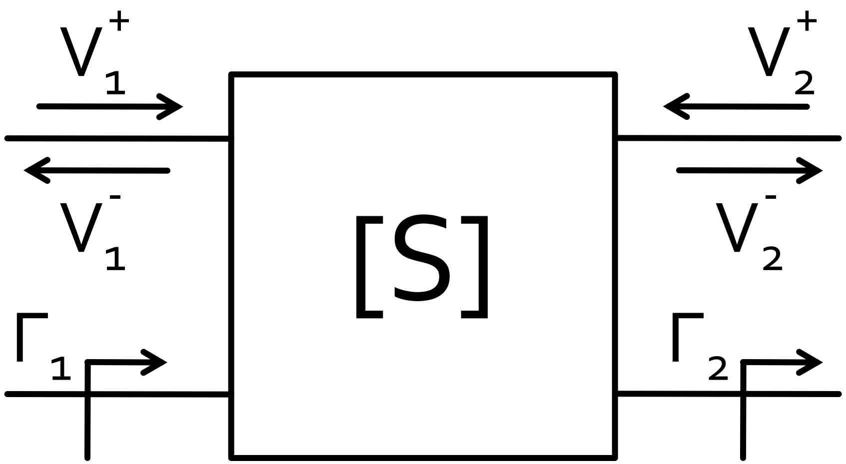
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} \qquad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \Gamma_1 \big|_{\Gamma_2=0}$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} = T_{21} \big|_{\Gamma_2=0}$$

- S11 is the reflection coefficient seen looking into port ***1*** when port ***2*** is terminated in matched load
- S21 is the transmission coefficient from port ***1*** (**second** index**!**) to port ***2*** (**first** index**!**) when port ***2*** is terminated in matched load

Scattering matrix – S

- S matrix can be extended to multiple ports

$$S_{ii} = \left.\frac{V_i^-}{V_i^+}\right|_{V_k^+=0,\forall k\neq i} \qquad S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,\forall k\neq j}$$

  - S_{ii} is the reflection coefficient seen looking into port *i* when all other ports are terminated in matched loads
  - S_{ij} is the transmission coefficient from port *j* (**second** index!) to port *i* (**first** index!) when all other ports are terminated in matched loads

Contact

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